Section 25
Integral

Integral is the sum of antiderivatives.

$\int_a^b f(x)dx = F(b) - F(a)$, $F(b) - F(a) -$ increase in antiderivative.

$F(b) - F(a) = F(x)\Big|_a^b$ $\int_a^b f(x)dx = F(x)\Big|_a^b$

$\int_a^b f(x)dx = F(b) - F(a)$ Newton-Leibniz formula.

$\int_a^b f(x)dx -$ formula for calculating the area of a curved figure.

Table integrals

$Sk \cdot dx = kx + C$, $k -$ number, $C -$ constant of integration.

$Sdx = x + c,$

$Sx^n dx = \frac{x^{n+1}}{n+1} + C, \ (n \neq -1).$

$S\frac{dx}{\sqrt{x}} = 2\sqrt{x} + C.$

$S\frac{dx}{x} = \ln|x| + C.$

$Se^x dx = e^x + C.$

$Sa^x dx = \frac{a^x}{\ln a} + C, \ (a > 0, \ a \neq 1).$

$\int \sin dx = -\cos x + C.$

$\int \cos dx = \sin x + C.$

$\int tgx dx = -\ln|\cos x| + C.$

$\int ctgx dx = \ln|\sin x| + C.$

$\int \frac{1}{\cos^2 x} dx = tgx + C.$

$\int \frac{1}{\sin^2 x} dx = -ctgx + C.$

$\int \frac{dx}{1+x^2} = arctgx + C.$

$\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C.$

$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C.$

$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C.$

Three rules for finding the integral:

1. $\int k\cdot f(x)dx = k\cdot\int f(x)dx.$
2. $\int\left(f(x)+\varphi(x)\right)dx = \int f(x)dx + \int\varphi(x)dx.$
3. $\int f(kx)dx = \frac{1}{k}\int f(kx)dx.$

$$\int_1^2 x^2dx = \frac{x^3}{3}\Big|_1^2 = \frac{2^3-1^3}{3} = \frac{7}{3} = 2\frac{1}{3}.$$

$$\int_\pi^{2\pi}\sin dx = -\cos x\Big|_\pi^{2\pi} = -\cos 2\pi - (\cos\pi) = -1+(-1) = -2.$$

$$\int_0^{\frac{\pi}{4}}\cos 2xdx = \frac{1}{2}\sin 2x\Big|_0^{\frac{\pi}{4}} = \frac{1}{2}\cdot\left(\sin\left(2\cdot\frac{\pi}{4}\right)-\sin(2\cdot 0)\right) = \frac{1}{2}\cdot(1-0) = \frac{1}{2}.$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 x} = -ctgx\Big|_{\frac{\pi}{3}}^{\frac{\pi}{3}} = -\left(ctg\frac{\pi}{3}-ctg\frac{\pi}{4}\right) = -\left(\frac{\sqrt{3}}{3}-1\right) = -\frac{\sqrt{3}-3}{3} = \frac{3-\sqrt{3}}{3}.$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right)dx = (tgx+ctgx)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left(tg\frac{\pi}{3}+ctg\frac{\pi}{3}\right)-\left(tg\frac{\pi}{6}+ctg\frac{\pi}{6}\right) =$$

$$= \left(\sqrt{3}+\frac{\sqrt{3}}{3}\right)-\left(\sqrt{3}+\frac{\sqrt{3}}{3}\right) = \sqrt{3}+\frac{\sqrt{3}}{3}-\frac{\sqrt{3}}{3}-\sqrt{3} = 0.$$

$$\int_{-2}^{-1}\left(x^{-3}-x\right)dx = \left(\frac{x^{-3+1}}{-3+1}-\frac{x^2}{2}\right)\Big|_{-2}^{-1} = \left(\frac{(-1)^{-2}}{-2}-\frac{(-1)^{-2}}{2}\right)-\left(\frac{(-2)^{-2}}{-2}-\frac{(-2)^{-2}}{2}\right) =$$

$$= \left(\frac{1}{-2}-\frac{1}{2}\right)-\left(\frac{1}{-8}-2\right) = -1-\left(-2\frac{1}{8}\right) = -1+2\frac{1}{8} = 1\frac{1}{8} = \frac{9}{8}.$$

$$\int_1^4\frac{x\cdot\sqrt[5]{x^2}}{\sqrt[10]{x^9}}dx = \int_1^4\frac{x\cdot x^{\frac{2}{5}}dx}{x^{\frac{9}{10}}} = \int_1^4 x^{1,4-0,9}dx = \int_1^4 x^{0,5}dx = \frac{x^{1,5}}{1,5}\Big|_1^4 = \frac{4^{1,5}}{1,5}-\frac{1^{1,5}}{1,5} = \frac{2^3-1}{1,5} = \frac{7}{1,5} = \frac{70}{15} = 4\frac{10}{15} = 4\frac{2}{3}.$$

$$\int_1^2\frac{dx}{2-3x} = \ln|2-3x|\cdot\left(-\frac{1}{3}\right)\Big|_1^2 = -\frac{1}{3}\cdot\left(\ln|2-3\cdot 2|-\ln|2-3\cdot 1|\right) = -\frac{1}{3}(\ln 4-\ln 1) = -\frac{1}{3}\ln 4.$$

$$\int_{-2}^{1}\left(x^2-2x+3\right)dx = \left(\frac{x^3}{3}-x^2+3x\right)\Big|_{-2}^{1} = 2\frac{1}{3}+\frac{8}{3}+10 = 15.$$

$$\int_1^3 e^{2x}dx = \frac{1}{2}\cdot e^{2x}\Big|_1^3 = \frac{1}{2}\cdot\left(e^{2\cdot 3}-^{2\cdot 1}\right) = \frac{1}{2}\cdot\left(e^6-^2\right) = \frac{e^2}{2}\cdot\left(e^4-1\right).$$

$$\int_{-1}^{3}\frac{dx}{\sqrt{2x+3}} = \frac{1}{2}\cdot 2\sqrt{2x+3}\Big|_{-1}^{3} = \sqrt{2x+3}\Big|_{-1}^{3} = \sqrt{2\cdot 3+3}-\sqrt{2\cdot(-1)+3} = \sqrt{9}-\sqrt{1} = 3-1 = 2.$$

$$\int_1^6\frac{dx}{\sqrt{x+3}} = 2\sqrt{x+3}\Big|_1^6 = 2\sqrt{6+3}-2\sqrt{1+3} = 2\cdot 3-2\cdot 2 = 6-4 = 2.$$

$$\int_{\frac{\pi}{2}}^{\pi}\cos\left(2x+\frac{\pi}{4}\right)dx=\frac{1}{2}\cdot\sin\left(2x+\frac{\pi}{4}\right)\Bigg|_{\frac{\pi}{2}}^{\pi}=\frac{1}{2}\cdot\left(\sin\left(2\pi+\frac{\pi}{4}\right)-\sin\left(2\cdot\frac{\pi}{2}+\frac{\pi}{4}\right)\right)=$$

$$=\frac{1}{2}\cdot\left(\sin\frac{\pi}{4}-\sin\left(\pi+\frac{\pi}{4}\right)\right)=\frac{1}{2}\cdot\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)=\frac{1}{2}\cdot\sqrt{2}=\frac{\sqrt{2}}{2}.$$

$$\int_{1}^{2}\left(3x^4+2x^5-5\right)dx=\left(3\cdot\frac{x^5}{5}+2\cdot\frac{x^3}{3}-5x\right)\Bigg|_1^2=\left(3\cdot\frac{2^5}{5}+2\cdot\frac{2^3}{3}-5\cdot 2\right)-\left(3\cdot\frac{1}{5}+2\cdot\frac{1}{3}-5\cdot 1\right)=$$

$$=\left(13\frac{3}{5}+5\frac{1}{3}-10\right)-\left(\frac{3}{5}+\frac{2}{3}-5\right)=19\frac{1}{5}+5\frac{1}{3}-10-\frac{3}{5}-\frac{2}{5}+5=18\frac{3}{5}+4\frac{2}{3}-5=13\frac{3}{5}+4\frac{2}{3}=$$

$$=17\frac{9+10}{15}=18\frac{4}{15}.$$

$$\int_{4}^{9}\left(\frac{2x}{5}+\frac{1}{2}\sqrt{x}\right)dx=\left(\frac{2}{5}\cdot\frac{x^2}{2}+\frac{1}{2}\cdot 2\sqrt{x}\right)\Bigg|_4^9=\left(\frac{x^2}{5}+\sqrt{x}\right)\Bigg|_4^9=\left(\frac{81}{5}+3\right)-\left(\frac{16}{5}+2\right)=13+1=14.$$

$$\int_{-4}^{2}\frac{xdx}{\sqrt{2-0{,}5x}}=\int_{-4}^{2}\frac{x-4+4}{\sqrt{2-0{,}5x}}dx=\int_{-4}^{2}\frac{x-4}{\sqrt{2-0{,}5x}}dx+\int_{-4}^{2}\frac{4dx}{\sqrt{2-0{,}5x}}=$$

$$=\int_{-4}^{2}\frac{-2\cdot(2-0{,}5x)}{\sqrt{2-0{,}5x}}dx+4\int_{-4}^{2}\frac{1}{\sqrt{2-0{,}5x}}dx=-2\int_{-4}^{2}\frac{\left(\sqrt{2-0{,}5x}\right)^2}{\sqrt{2-0{,}5x}}dx+4\int_{-4}^{2}\frac{1}{\sqrt{2-0{,}5x}}dx=$$

$$=-2\int_{-4}^{2}\sqrt{2-0{,}5x}dx+4\cdot 2\sqrt{2-0{,}5x}\cdot(-2)\Big|_{-4}^{2}=-2\cdot(2-0{,}5x)^{\frac{3}{2}}\cdot\frac{2}{3}\cdot(-2)\Big|_{-4}^{2}-16\cdot(2-0{,}5x)^{\frac{1}{2}}\Big|_{-4}^{2}=$$

$$=\frac{8}{3}\cdot(1-8)-16\cdot(1-2)=-\frac{56}{3}+16=-\frac{8}{3}.$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}tg^2x=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\sin^2 x}{\cos^2 x}dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{1-\cos^2 x}{\cos^2 x}dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\left(\frac{1}{\cos^2 x}-1\right)dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{1}{\cos^2 x}dx-\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}1\cdot dx=tgx\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}-x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}=$$

$$=tg\frac{\pi}{4}-tg\frac{\pi}{6}-\frac{\pi}{4}+\frac{\pi}{6}=1-\frac{\sqrt{3}}{3}-\frac{\pi}{12}.$$

One way to calculate the integrals is the introduction of a new variable,.

$$\int_{-\frac{\pi}{6}}^{0}\frac{dx}{\cos^2\left(2x+\frac{\pi}{3}\right)}=$$

$$2x+\frac{\pi}{3}=t,\ 2\cdot\left(-\frac{\pi}{6}\right)+\frac{\pi}{3}=t,\ -\frac{\pi}{3}+\frac{\pi}{3}=t,$$

$t=0,$

$t=\frac{\pi}{3}$ - integration limits.

$$2\cdot 0+\frac{\pi}{3}=t,\ 2x=t-\frac{\pi}{3}\Big|:2\ \ x=\frac{1}{2}t-\frac{\pi}{6},\ dx=\frac{1}{2}dt-0,\ dx=\frac{1}{2}dt.$$

$$\int_{0}^{\frac{\pi}{3}}\frac{\frac{1}{2}dt}{\cos^2 t}=\frac{1}{2}\int_{0}^{\frac{\pi}{3}}\frac{dt}{\cos^2 t}=\frac{1}{2}tgt\Big|_0^{\frac{\pi}{3}}=\frac{1}{2}\left(tg\frac{\pi}{3}-tg0\right)=\frac{1}{2}\cdot\frac{\sqrt{3}}{1}=\frac{1}{2}\cdot 0=\frac{\sqrt{3}}{2}.$$

Calculating the areas of shapes using a definite integral

When calculating the areas of figures using the integral, it is appropriate to adhere to the following sequence of actions (steps):
1) create a system of equations from the conditions of the problem and solve it;
2) taking into account the conditions of the problem and the found solutions of the system of equations, perform sketches of the graphs of the functions encountered in the problem.
3) using a sketch to set which of the functions is decreasing (and the covering sketch on top) and which is negative (graph below);
4) define the boundaries of integration;
5) calculate the value of the integral.

A task. Calculate the area of a shape bounded by lines $y = x^2$ and $y = x^3$.

Solution:

$$\begin{cases} y = x^2, \\ y = x^3. \end{cases}$$

$x^3 = x^2,$

$x^3 - x^2 = 0,$

$x^2(x-1) = 0,$

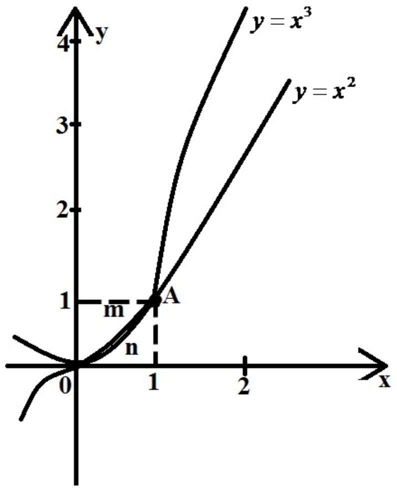
$x_1 = 0,\ \ x - 1 = 0,\ \ x_2 = 1.$

$y_1 = 0,$

$y_2 = 1.$

0 and 1 - integration limits.

$(0;0)$ и $(1;1)$ - intersection points of function graphs.



$$S_{OmAn} = \int_0^1 \left(x^2 - x^3\right)dx =$$

$$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = \left(\frac{1}{3} - \frac{1}{4}\right) - 0 = \frac{4-3}{12} = \frac{1}{12}\ (кв.од)$$

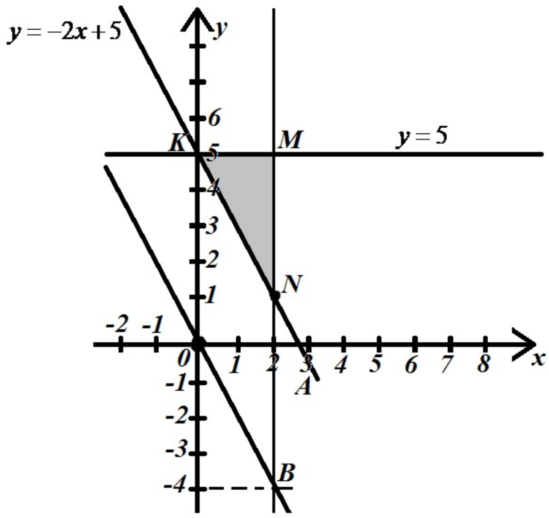
Answer: $\frac{1}{12}$ $(кв.од)$.

When the figure is fully or partially located below the abscissa axis, then you can swap the limits of integration or put the "-" sign in front of the integral to the interval in which the figure is below the *OX* axis.
Both of these methods are not without their drawbacks. In this situation, you can use the following method: carry out a parallel transfer of the figure, the area of which must be calculated along the *OY* axis to such a distance that the figure is completely located above the *OX* axis.
Such a transformation of the plane means that the functions data in the conditions of the problem are replaced by other functions formed by adding functions of the same number to the data.
A task. Calculate the area of a shape bounded by lines $y=-2x,\ x=2,\ y=0.$

Solution:

$$\begin{cases} y=-2x, & -2x=0, \\ y=0, & x_1=0, \\ x=2. & x_2=2. \end{cases}$$ 0 and 2 - integration limits.

$y_1=0;\ y_2=-2\cdot 2=-4.\ (0;\ 0),\ (2;\ -4)-$ system solutions.



It is necessary to calculate the area of the triangle *OAB*, which is located under the *OX* axis.
Add to both functions $y=0$ and $y=-2x$ number *5*: we get new functions $y=0+5=5;\ y=-2x+5.$
Let us construct graphs of these functions and find the area of the triangle *KMN* equal to the area of the triangle *OAB*. Integration limits are preserved.

$$S_{\Delta KMN}=\int_0^2 5dx-\int_0^2(-2x+5)dx=5\cdot x\Big|_0^2+2\int_0^2 xdx-5x\Big|_0^2=2\cdot\frac{x^2}{2}\Big|_0^2=2^2-0^2=4\ (\textit{кв.од})$$

Answer: 4 *(square unit)*.
A task. Calculate the area of a shape bounded by lines $y=3x-x^2,\ y=0,\ x=4.$

Solution:

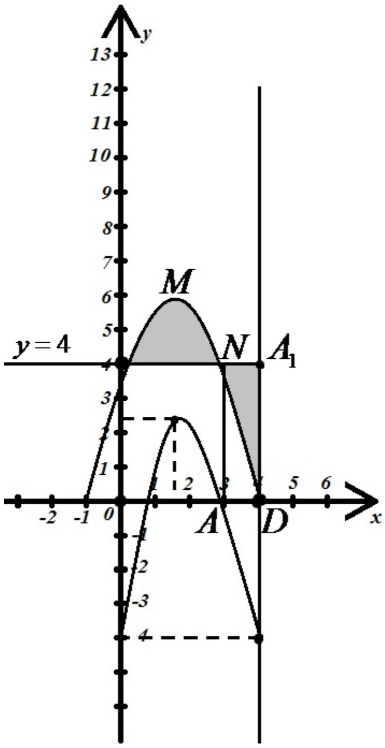
$$\begin{cases} y=3x-x^2, & 3x-x^2=0, & 3-x=0, & y_1=3\cdot 0-0^2=0, \\ y=0, & x(3-x)=0, & x_2=3, & y_2=3\cdot 3-3^2=0, \\ x=4. & x_1=0. & x_3=4. & y_3=3\cdot 4-4^2=-4. \end{cases}$$

$0\,(0;0);\ A\,(3;0);\ B\,(4;-4).$
We find the coordinates of the vertex of the parabola:

$m=-\frac{b}{2a}=-\frac{3}{2\cdot(-1)}=1{,}5;\ \ n=y(m)=3\cdot 1{,}5-1{,}5^2=4{,}5-2{,}25=2{,}25.$

$(1{,}5;\ 2{,}25)$. Lowest point of a shape - $B(4;-4)$.

Perform a parallel transfer of the figure 4 units up.



$S_{A_1DNMK}=S_{NMK}+S_{A_1BN_1}$,

$S_{NMK}=\int_0^3\left(3x-x^2+4\right)dx=\int_0^3 4dx=3\cdot\frac{x^2}{2}\Big|_0^3-\frac{x^3}{3}\Big|_0^3+4x\Big|_0^3-4x\Big|_0^3=$

$=\frac{3}{2}\cdot(9-0)-\frac{1}{3}\cdot(27-0)+4\cdot(3-0)-4\cdot(3-0)=\frac{27}{2}-9+12-12=13{,}5-9=4{,}5\ (\text{кв.од})$

$S_{NA_1D}=\int_3^4\left(4-\left(3x-x^2\right)\right)dx=\int_3^4\left(4-3x+x^2\right)dx=\int_3^4 4dx-\int_3^4 3xd+\int_3^4 x^2dx=4x\Big|_3^4-3\cdot\frac{x^2}{2}\Big|_3^4+\frac{x^3}{3}\Big|_3^4=$

$=4\cdot(4-3)-\frac{3}{2}\cdot(16-9)+\frac{1}{3}\cdot(64-27)=4-\frac{21}{2}+\frac{37}{3}=\frac{12+63+74}{6}=\frac{86-63}{6}=\frac{23}{6}=3\frac{5}{6}.$

$S_{A_1DNMK}=4\frac{1}{2}+3\frac{5}{6}=7\frac{3+5}{6}=7\frac{8}{6}=8\frac{2}{6}=8\frac{1}{3}\ (\text{кв.од})$

Answer: $8\frac{1}{3}\ (\text{кв.од})$

The text of the augmented problem.

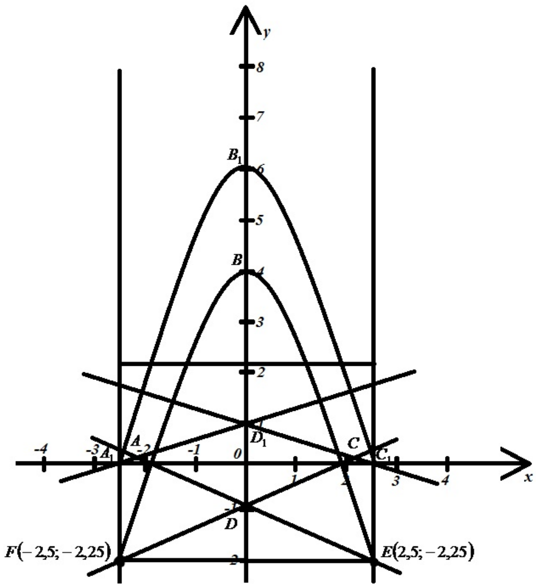
Calculate the area of a shape: $y=4-x^2;\ \ y=-\frac{1}{2}x-1,\ \ y=\frac{1}{2}x-1.$

Solution:

We find the limits of integration:

$$\begin{cases} y=4-x^2, \\ y=-\frac{1}{2}x-1 \end{cases}\quad \begin{matrix} 4-x^2=-\frac{1}{2}x-1, \\ 4-x^2+\frac{1}{2}x+1=0\,|\cdot 2 \end{matrix}\quad \begin{matrix} 8-2x^2+x+2=0, & 2x^2-x-10=0, \\ -2x^2+x+10=0 & D=1+80=81. \end{matrix}$$

$$x_1 = \frac{1-\sqrt{81}}{4} = \frac{1-9}{4} = \frac{-8}{4} = -2. \quad x_2 = \frac{1+9}{4} = \frac{10}{4} = 2{,}5.$$



$$S_1 = \int_{-2,5}^{2,5} \left(6{,}25 - x^2\right)dx = \left(6{,}25x - \frac{x^3}{3}\right)\Big|_{-2,5}^{2.5} = \left(15{,}625 - \frac{15{,}625}{3}\right) - \left(-15{,}625 - \frac{-15{,}625}{3}\right) =$$

$$= 31{,}25 - \frac{31{,}25}{2} = \frac{31{,}25}{2} = 15{,}625\ (кв.од)$$

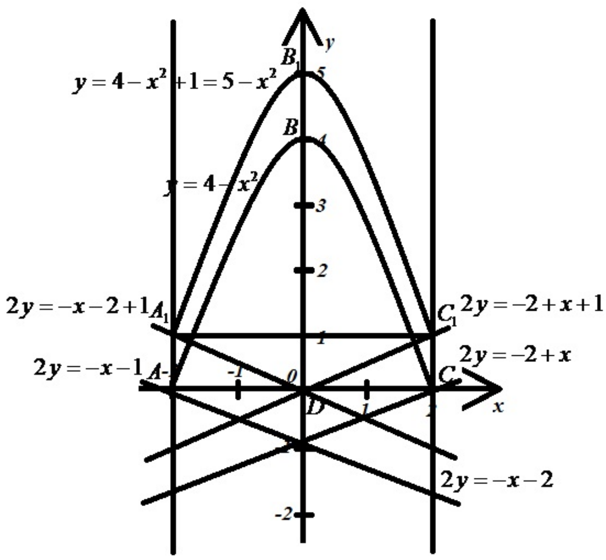
$$S_2 = S_{\Delta AD_1C} = \frac{1}{2} \cdot A_1C_1 \cdot OD_1 = \frac{1}{2} \cdot (2{,}5 + 2{,}5) \cdot 1{,}25 = 2{,}5 \cdot 1{,}25 = 3{,}125.$$

$$S_Ф = 15{,}625 - 3{,}125 = 12{,}5\ (кв.од)$$

Calculate the area of a shape bounded by lines:

$x = -2,\ \ x = 2,\ \ y = 4 - x^2,\ \ 2y = -x - 2,\ \ 2y = x - 2.$

Solution:



Move the *ABCD* figure one unit up along the *OY* axis.

A figure formed $A_1B_1C_1D_1$.

We consider the area of the desired figure as the sum of the areas of the figures A_1B_1O and OB_1C_1

$$S_{A_1B_1O} = \int_{-2}^{0}\left(5 - x^2 - \left(-\frac{1}{x} - 1\right)\right)dx = \int_{-2}^{0}\left(-x^2 + \frac{1}{2}x + 6x\right)dx = \left(-\frac{x^3}{3} + \frac{x^2}{4} + 6x\right)\Big|_{-2}^{0} =$$

$$= 0 - \left(-\frac{-8}{3} + 1 + 12\right) = 6\frac{1}{3}\ (кв.од)$$

$$S_{OB_1C_1} = \int_{0}^{2}\left(5 - x^2 - \left(-\frac{1}{x} + 1\right)\right)dx = \int_{0}^{2}\left(-x^2 - \frac{1}{2}x + 6x\right)dx = \left(-\frac{x^3}{3} - \frac{x^2}{4} + 6x\right)\Big|_{0}^{2} =$$

$$= -\frac{8}{3} - 1 + 12 = -5\frac{2}{3} + 12 = 6\frac{1}{3}\ (кв.од)$$

$$S_{Ф} = 6\frac{1}{3} + 6\frac{1}{3} = 12\frac{2}{3}.$$

Answer: $12\frac{2}{3}$.

Find the area of the figure bounded by lines $y = -x^2 + 4x - 3$, $y = -x^2 + 6x - 5$, $y = 3x - 15$.

Solution:

We find the limits of integration:

$$\begin{cases} y = -x^2 + 6x - 5, \\ y = -x^2 + 4x - 3. \end{cases} \quad -x^2 + 6x - 5 = -x^2 + 4x - 3, \quad 6x - 4x = -3 + 5, \ 2x = 2.$$

$x_1 = 1$, $y_1 = 0$ intersection point of two parabolas.

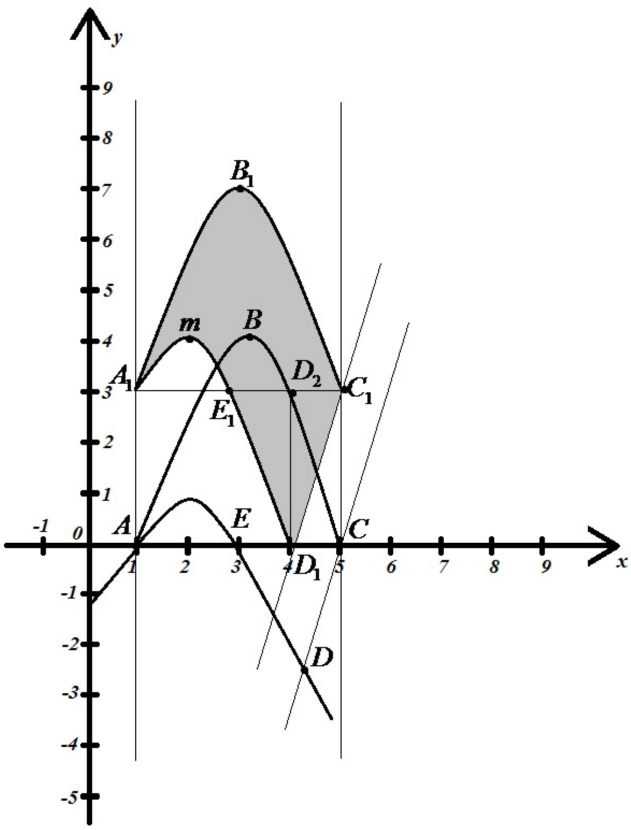
$$\begin{cases} y = -x^2 + 6x - 5, \\ y = 3x - 15. \end{cases} \quad -x^2 + 6x - 5 = 3x - 15, \quad -x^2 + 6x - 5 - 3x + 15 = 0, \quad -x^2 + 3x + 10 = 0. \quad x^2 - 3x - 10 = 0.$$

$D = 9 + 40 = 49$, $x_1 = \frac{3-7}{2} = -\frac{4}{2} = -2$; $y_1 = -21$. $x_2 = \frac{3+7}{2} = \frac{10}{2} = 5$, $y_2 = 0$ intersection points of a straight line with a parabola $y = -x^2 + 6x - 5$.

$$\begin{cases} y = -x^2 + 4x - 3, \\ y = 3x - 15. \end{cases} \quad -x^2 + 4x - 3 = 3x - 15, \quad -x^2 + x + 12 = 0. \quad x^2 - x + 12 = 0.$$

$x_1 = 4$, $x_2 = -3$, $y_1 = -3$. $y_2 = -25$. intersection points of a straight line with a parabola $y = -x^2 + 4x - 3$.

Let's sketch.



We find the coordinates of the vertices of the parabolas:

$y=-x^2+6x-5,\quad x_0=-\dfrac{6}{-2}=3,\quad y_0=-9+18-5=4,\ (3;4)$

$y=-x^2+4x-3.\quad x_0=\dfrac{-4}{-2}=2.\quad y_0=-4+8-3=1.\ (2;1)$

Move the ABCDE figure up along the ordinate axis by 3 units.

$-x^2+6x-5+3=-x^2+6x-2.$

$$S_{A_1B_1C_1}=\int_1^5\left(-x^2+6x-2-3\right)dx=\left(-\frac{x^3}{3}+\frac{6x^2}{2}-5x\right)\Big|_1^5=-\frac{125}{3}+3\cdot 25-25-\left(-\frac{1}{3}+3\cdot 1-5\right)=$$

$$=-\frac{125}{3}+50+\frac{1}{3}-3+5=-\frac{124}{3}+52=-41\frac{1}{3}+52=10\frac{2}{3}\,(кв.од)$$

$-x^2+4x-3+3=-x^2+4x.$

$$S_{A_1m_1E_1}=\int_1^3\left(-x^2+4x-3\right)dx=\left(-\frac{x^3}{3}+2x^2-3x\right)\Big|_1^3=-9+18-9-\left(-\frac{1}{3}+2-3\right)=1\frac{1}{3}\,(кв.од)$$

$$S_{A_1B_1C_1E_1}=10\frac{2}{3}-1\frac{1}{3}=9\frac{1}{3}\,(кв.од)$$

$$S_{ED_2D_1}=\int_3^4\left(3+x^2-4x\right)dx=\left(3x+\frac{x^3}{3}-4\right)\Big|_3^4=\left(12+\frac{64}{3}-2\cdot 16\right)-\left(9+9-2\cdot 9\right)=1\frac{1}{3}\,(кв.од)$$

$$S_{D_1D_2C_1}=\int_4^5\left(3-3x+15-3\right)dx=\int_4^5\left(15-3x\right)dx=\left(15x-\frac{3x^2}{2}\right)\Big|_4^5=\left(75-\frac{75}{2}\right)-\left(60-24\right)=$$

$$=37\frac{1}{2}-36=1\frac{1}{2}\,(кв.од)$$

$$S_{\Phi}=9\frac{1}{3}+1\frac{1}{3}+1\frac{1}{2}=11\frac{2}{3}+\frac{1}{2}=11\frac{4+3}{6}=12\frac{1}{6}\,(кв.од)$$

Self-study assignments:

$\int_{-1}^{1} x^4 dx.$ Answer: 0,4.

$\int_{-2}^{2} x^3 dx.$ Answer: 0.

$\int_{-\pi}^{0} \sin x dx.$ Answer: $-2.$

$\int_{-3}^{0} 4x^3 dx.$ Answer: $-81.$

$\int_{-3}^{2} (2x-3) dx.$ Answer: $-20.$

$\int_{-2}^{-1} (5-4x) dx.$ Answer: 11.

$\int_{-1}^{2} (1-3x^2) dx.$ Answer: $-6.$

$\int_{4}^{9} \left(3x - \frac{4}{\sqrt{x}}\right) dx.$ Answer: 89,5.

$\int_{-1}^{1} (x+1)^2 dx.$ Answer: $2\frac{2}{3}.$

$\int_{0}^{2} e^{3x} dx.$ Answer: $\frac{1}{3} \cdot (e^6 - 1).$

$\int_{0}^{\frac{\pi}{4}} \cos 4x dx.$ Answer: 0.

$\int_{\frac{\pi}{4}}^{\pi} \cos(3x - 45^\circ) dx.$ Answer: $\frac{\sqrt{2}-2}{6}.$

$\int_{-1}^{1} (x^3 + 2x - 1) dx.$ Answer: $-2.$

$\int_{-2}^{1} (2x^3 + 3x - 4) dx.$ Answer: $-24.$

$\int_{1}^{2} (5-2x) dx.$ Answer: 2.

$\int_{1}^{4} \left(1 + 5x + \frac{3\sqrt{x}}{2}\right) dx.$ Answer: 47,5.

$\int_{-1}^{0} \sqrt{3-5x} dx.$ Answer: $\frac{2}{15}\left(16\sqrt{2} - 3\sqrt{3}\right).$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} ctg^2 2x dx.$ Answer: $\frac{1}{2\sqrt{3}} - \frac{\pi}{12}.$

Calculate the area of a shape bounded by lines:

а) $y = -x^2 - 2x + 3$ and $y = 1 - x.$ Answer: $4{,}5\,(кв.од)$

б) $2y = x^2 + x - 6$ and $2y = -x^2 + 3x + 6.$ Answer: $20\frac{5}{6}\,(кв.од)$

1) $x = 0,\ y = 0,\ y = \sin x.$ Answer: $2\,(кв.од)$

2) $x = 0,\ x = 4, y = 0, y = 3x^2 - 6.$ Answer: $24\,(кв.од)$

3) $y = 0,\ y = 2x^2 + 3x - 9.$ Answer: $30\frac{3}{8}\,(кв.од)$

4) $y = x^2 + 6x + 5,\ y = 0.$ Answer: $10\frac{2}{3}\,(кв.од)$

5) $y = 0,\ y = x^3 - 6x^2 + 11x - 6.$ Answer: $0{,}5\,(кв.од)$

6) $y = 0,\ y = (x+2)^2,\ y = 4 - x.$ Answer: $10\frac{2}{3}\,(кв.од)$

Calculate integrals:

$\int_0^4 \frac{x^2 - \sqrt[3]{x^2}}{\sqrt{x}} dx.$ Answer: $12\frac{4}{5} - 3\frac{3}{7}\sqrt[3]{2}.$

$\int_0^{\frac{\pi}{4}} \left(\frac{2}{\cos^2 x} + \sin x \right) dx.$ Answer: $\frac{6 - \sqrt{2}}{2}.$

For function $f(x) = x^2 - 2x + 1$ find that original $F(x)$, the graph of which passes through the point $M(2;1)$.

Answer: $F(x) = \frac{1}{3}x^3 - x^2 + x + \frac{1}{3}.$

Calculate the area of a shape bounded by lines $y = x^2 - 6x + 5$ and coordinate axes.

Answer: $2\frac{1}{3}\,(кв.од).$

Calculate the area of a shape bounded by lines $y = x^2 + 2x - 2$ and $y = x.$

Answer: $4{,}5\,(кв.од).$