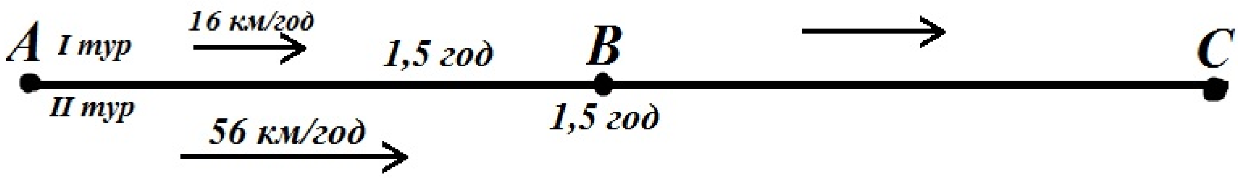
Section 22
Solving word problems using equations and their systems

Tasks for movement.

№ 13.079 The first tourist, having ridden a bicycle for 1.5 *hours* at a speed of 16 *km/h*, makes a stop for 1.5 *hours*, and then continues to move at an initial speed. 4 *hours* after the departure of the first tourist, the second tourist leaves behind him on a motorcycle at a speed of 56 *km/h*.
How far will they travel to the place where the second tourist will catch up with the first?

Solution:



Let the second tourist catch up with the first at point C, located *x km* away from point A.

$\frac{x}{16}$ *h* - time for the first tourist to cover *x km*.

$\frac{x}{56}$ *h* - time for the second tourist to cover *x km*.

Since the first tourist, having driven for 1.5 hours, made a stop at point B for 1.5 hours, the time of his movement from A to C on $4-1{,}5=2{,}5$ *h* more from the time of movement of the second tourist from A to C.
According to the condition of the problem, we compose the equation:

$$\frac{x}{16}-\frac{x}{56}=2{,}5;\ \frac{56x-16x}{16\cdot 56}=\frac{5}{2};\ \frac{40x}{896}=\frac{5}{2}.$$

Taking advantage of the main property of proportion:

$$40x\cdot 2=896\cdot 5;\ x=\frac{4480}{80}=56.$$

Answer: 56 *km*.

When solving problems using equations (their systems), it is appropriate to adhere to such a scheme:

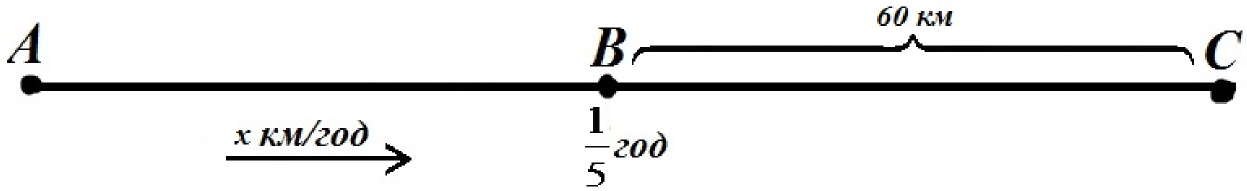
1) Divide the task into *conditions* and *requirements*;
2) Unknown (unknown's) quantity (s) shall be denoted by a variable (s);
3) Using the conditions of the problem, link known and unknown quantities in the equation (in equations);
4) Solve equation (system of equations);

5) From the solutions of an equation or a system of equations, choose the answer to the problem.

№ 13.083 The train was delayed on the way for 12 *minutes*, and then, at a distance of 60 km, made up for the lost time, increasing its speed by 15 *km/h*.
Find the starting speed of the train.

Solution:

$12\ min = \frac{12}{60} ч = \frac{1}{5} ч.$



Let *x km/h* - the initial speed of the train.
At this rate, the train moved from point A to point B.
$\frac{1}{5}$ *h* he stood at point B. On the BC = 60 *km* section, the train was moving at a speed $(x+15)$*km/h*, making up for the lost $\frac{1}{5}h$ in point B.
If the train did not increase its initial speed on the BC section, it would have overcome it in $\frac{60}{x}$ *h*. By increasing the speed by 15 *km/h*, he spent on BC $\frac{60}{x+15}$ *h*.
The time difference is $\frac{1}{5}$h.
This is the basis for the compilation of the equation:

$$\frac{60}{x} - \frac{60}{x+15} = \frac{1}{5};\ x \neq 0, x \neq -15.$$

$$\frac{60x + 60 \cdot 15 - 60x}{x \cdot (x+15)} = \frac{1}{5};\ \frac{900}{x \cdot (x+15)} = \frac{1}{5};$$

By the main property of the proportion, we have: $x(x+15) = 4500;\ x^2 + 15x4500 = 0;$
By Vieta's theorem: $x_1 = 60;\ x_2 = -75.$
Since the speed is an unknown quantity, the root -75 does not satisfy the condition of the problem.
Answer: 60 *km/h*.

№ 13.101 Simultaneously, two pedestrians were to go out to meet each other of the two points. Pedestrian A from village M, pedestrian B - from village K.
But pedestrian A was delayed and left 6 *hours* later. Upon meeting, it turned out that pedestrian A walked 12 km less than pedestrian B. Having rested, they simultaneously left the meeting place and continued to move at the same speeds.
As a result, A came to K in 8 *hours*, and B came to M in 9 *hours* after the meeting.
Determine MK distance and speed of pedestrians.

Solution:

Let x *km/h* be the speed of the pedestrian A, y *km/h* the speed of the pedestrian B. C - meeting point.

After the meeting, pedestrian A walked the path CK = $8x$ *km*, pedestrian B passed CM = $9y$ *km*. Because $MC<CK$ of 12, then we have the equation: $8x-9y=12\,(1)$

Pedestrian A spent on the path MC $\frac{9y}{x}$ *h*, and a pedestrian B on the way KC spent $\frac{8x}{y}$ *h*. Since the difference in the time spent on the way of both pedestrians is 6 hours, then on this basis we compose the equation:

$\frac{8x}{y}-\frac{9y}{x}=6\,(2)$ The values of the variables x and y must satisfy the equations (1) и (2), and therefore they must satisfy such a system:

$\begin{cases}8x-9y=12,\\ \frac{8x}{y}-\frac{9y}{x}=6.\end{cases}$ Let the $\frac{x}{y}=t$, then $\frac{y}{x}=\frac{1}{t}$; $x=ty$. $t>0$, because $x>0$, $y>0$.

$\begin{cases}8x-9y=12,\\ 8t-\frac{9}{t}=6.\end{cases}$ $\begin{cases}8x-9y=12,\\ \frac{8t^2-6t-9}{t}=0.\end{cases}$ $\begin{cases}8x-9y=12,\\ 8t^2-6t-9=0.\end{cases}$

$D=36+288=324=18^2,$

$t_1=\frac{6-18}{16}=-\frac{12}{16}=-\frac{3}{4}$ – does not satisfy the condition of the problem, since $x>0, y>0.$

$t_2=\frac{6+18}{16}=\frac{24}{16}=\frac{3}{2}=1{,}5.$ $x=ty=1{,}5y.$ Substituting into the first equation of the system, we obtain: $8\cdot 1{,}5y-9y=12;$ $12y-9y=12;$ $3y=12;$ $y=\frac{12}{3}=4.$ $x=1{,}5\cdot 4=6.$

$MK=9\cdot 4+8\cdot 6=3+48=84$ *(km)*.

Answer: 6 *km/h*, 4 *km/h*, 84 *km*.

№ 13.129 At 9 o'clock the self-propelled barge left point A upriver and arrived at point B. After staying for 2 hours at point B, she went to point A, which she reached at 19.20 on the same day. River flow speed 3 *km/h*. Distance from A to B is 60 *km*. Barge speed - constant.

What time the barge arrived at the point B.

Solution:

9 *h* 20 *min* = $19\frac{1}{3}$ *h*.

Let the x *km/h* own speed of the barge, then

$(x+3)$ *km/h* - barge speed downstream,

$(x-3)$ *km/h* - barge speed upstream,

$\frac{60}{x+3}$ *h* - time to go with the flow,

$\frac{60}{x-3}$ *h* - time to move against the flow.

Total time spent $19\frac{1}{3}-(9+2)=8\frac{1}{3}$ *(ч)*.

According to the condition of the problem, we compose the equation:

$\frac{60}{x+3}+\frac{60}{x-3}=8\frac{1}{3};\ \frac{60}{x+3}+\frac{60}{x-3}=\frac{25}{3}\Big|:5\ \ \frac{12}{x+3}+\frac{12}{x-3}=\frac{5}{3};\ x\neq-3;\ x\neq3;$

$\frac{12x-36+12x+36}{(x+3)\cdot(x-3)}=\frac{5}{3};\ \frac{24}{x^2-9}=\frac{5}{3};$

Let's use the main property of proportion:

$5x^2-72x-45=0:5\cdot(x^2-9)=3\cdot24x;\ D=72^2+900=5184+900=6084=78^2.$

$x_1=\frac{72-78}{10}=-0{,}6-$ does not satisfy the condition of the problem.

$x_2=\frac{72+78}{10}=15.$

15 *km/h* - own speed of the barge. To point B, the barge moved against the current, spending $\frac{60}{15-3}=\frac{60}{12}=5$ *(h)*.

Since she left point A at 9 o'clock, she arrived at point B at $9+5=14$ *(h)*.

Answer: 14 *h*.

№ 13.278 The passenger of the train knows that on this section of the track, the speed of this train is 40 *km/h*. As soon as the oncoming train began to pass the window, the passenger turned on the stopwatch and noticed that the oncoming train was passing the window for 3 seconds.

Determine the speed of the oncoming train if its length 75 *m*.

Solution:

Let the *x km/h* - oncoming train speed.

3 *sec* = $\frac{3}{3600}h=\frac{1}{1200}\ h.$

During this time, the train with passengers will pass $40\cdot\frac{1}{1200}=\frac{1}{30}$ *km*, and the oncoming train $\frac{1}{1200}x$ *km*.

Only two trains have passed 75 *m* = $\frac{75}{1000}$ *km* = $\frac{3}{40}$ *km*.

According to the condition of the problem, we compose the equation:

$\frac{x}{1200}+\frac{1}{30}=\frac{3}{40};\ \frac{x}{1200}=\frac{3}{40}-\frac{1}{30};\ \frac{x}{1200}=\frac{90-40}{1200};\ \frac{x}{1200}=\frac{50}{1200};\ x=50.$

Answer: 50 *km/h*.

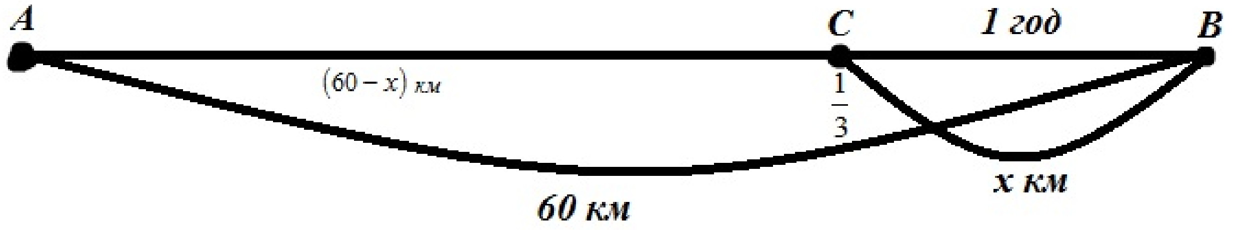
№ 13.360 The distance between points A and B is 60 *km*. The cyclist rode from A to B. Without stopping at B, he went to A with the same speed. 1 hour after leaving point B, he makes a stop for 20 minutes. After that, he continues to move, increasing the speed by 4 *km/h*.

What are the limits of the cyclist's speed if he spent no more than A to B on the way back from B to A.

Solution:

20 *min* = $\frac{1}{3}$ *h*.

x km/h - cyclist's initial speed.



Then $\frac{60}{x}$ *h* - time spent on path AB. The time spent at BC and stopping at point C is: $1+\frac{1}{3}=1\frac{1}{3}$ *(h)*.

The BC path is *x km*, then the CA path is 60*x km*. Since after stopping at point C on $\frac{1}{3}$ *h* the cyclist increased his speed by 4 *km/h*, then on the way the CA was moving at a speed $(x+4)$ *km/h*.

The time spent on the path of the CA: $1\frac{1}{3}+\frac{60-x}{x+4}$.

According to the condition of the problem, the time spent on the path from B to A is no more than from A to B, then we can compose the following inequality:

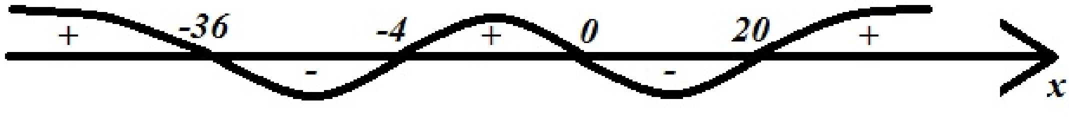
$1\frac{1}{3}+\frac{60-x}{x+4}\le\frac{60}{x}$; $1\frac{1}{3}+\frac{60-x}{x+4}-\frac{60}{x}\le 0$; $\frac{4}{3}+\frac{60-x}{x+4}-\frac{60}{x}\le 0$;

$\frac{4x\cdot(x+4)+(60-x)\cdot 3x-60\cdot 3\cdot(x+4)}{3x\cdot(x+4)}\le 0$; $\frac{4x^2+16x+180x-3x^2-180x-720}{3x\cdot(x+4)}\le 0$;

$\frac{x^2+16x-720}{3x\cdot(x+4)}\le 0; (A)$

We expand the quadratic trinomial $x^2+16x-720$ factoring, solving the equation $x^2+16x-720=0$. By Vieta's theorem $x_1=-36;\ x_2=20$. Then, $x^2+16x-720=(x+36)\cdot(x-20)$ and inequality A takes the form $\frac{(x+36)(x-20)}{3x(x+4)}\le 0$.

We solve this inequality by the *"snake"* method:



$x\in[-36;-4)\cup(0;20]$.

Since the speed is not negative, then $[-36;-4)$ does not satisfy the condition of the problem.

Answer: $x\in(0;20]$ *km/h*.

Task. Two pohohod went out at the same time towards each other and met after 3 hours 20 minutes.
How long does it take for each of them to travel the entire distance if the first arrives at that point later than the second arrived at the point from which the first left?

Solution:

Let the 1 - all distance, *x h* – time of the first pedestrian, and *y h* – time of the second pedestrian.

$\frac{1}{x}$ *km/h* - speed of the first pedestrian,

$\frac{1}{y}$ *km/h* - speed of the second pedestrian.

3 h 20 min $= 3\frac{1}{3}$ *h.*

$\frac{1}{x}\cdot 3\frac{1}{3}$ *(km)* - path of the first pedestrian before the meeting.

$\frac{1}{y}\cdot 3\frac{1}{3}$ *(km)* - path of the second pedestrian before the meeting.

According to the conditions of the problem, we compose the system of equations:

$$\begin{cases} x - y = 5, \\ \frac{1}{x}\cdot 3\frac{1}{3} + \frac{1}{y}\cdot 3\frac{1}{3} = 1. \end{cases} \quad \begin{cases} x = 5 + y, \\ \frac{10}{3x} + \frac{10}{3y} = 1. \end{cases} \quad \begin{cases} x = 5 + y, \\ \frac{30y + 30x}{9xy} = 1. \end{cases}$$

$$x \neq 0, \;\; y \neq 0. \;\; \begin{cases} x = 5 + y, \\ 30y + 30x = 9xy \,|:3 \end{cases} \quad \begin{cases} x = 5 + y, \\ 10y + 10x = 3xy. \end{cases}$$

$10y + 10\cdot(5 + y) = 3y\cdot(5 + y),\;\; 10y + 50 + 10y = 15y + 3y^2,\;\; 3y^2 + 15y - 20y - 50 = 0,$

$3y^2 - 5y - 50 = 0,\;\; D = 25 + 600 = 625 = 25^2;$

$y_1 = \frac{5 - 25}{6} = -\frac{20}{6}$ - does not satisfy the condition of the problem.

$y_2 = \frac{5 + 25}{6} = 5;\;\; x_2 = 5 + 5 = 10.$

Answer: *10 h* - travel time of the first pedestrian; *5 h* - travel time of the second pedestrian.

Task. Two cyclists rode out from two points in the third, agreeing to come to him at the same time. The first one arrived at the meeting point via *2 h*, another, in order to arrive on time, had to drive every kilometer for *1 min* faster than the first, since its path was longer by *6 km*.
What is the speed of each cyclist?

Solution:

Let the first cyclist covered one kilometer in x minutes, then his speed $\frac{1}{x}\frac{км}{хв}=\frac{1}{x}\frac{км}{\frac{1}{60}год}=\frac{60}{x}\frac{км}{год}$. The second cyclist rode 1 *km* 1 *min* faster, that is, $(x-1)$ *min*. His speed $\frac{1}{x-1}\frac{км}{хв}=\frac{1}{x-1}\frac{км}{\frac{1}{60}год}=\frac{60}{x-1}\frac{км}{год}$.

$\frac{60}{x}\cdot 2$ *(km)* - passed the first cyclist for 2 *h*. Since the difference between their paths is 6 *km*, then we compose the equation:

$\frac{60}{x-1}\cdot 2-\frac{60}{x}\cdot 2=6|:6\ \ \frac{10}{x-1}\cdot 2-\frac{10}{x}\cdot 2=1;\ \ \frac{20}{x-1}-\frac{20}{x}=1;\ \ \left.\begin{matrix}x\neq 1,\\ x\neq 0.\end{matrix}\right\}-$ *range of valid values*

$\frac{20x-20x+20}{(x-1)\cdot x}=1.$

$20=(x-1)\cdot x,\ 20=x^2-x,\ x^2-x-20=0.$ By Vieta's theorem $x_1=5,\ x_2=-4-$ the roots of a quadratic equation. Since the time - additional value, the number of $-4-$ does not satisfy the condition of the problem.

$\frac{60}{5}=12$ *km/h* - speed of the first cyclist.

$\frac{60}{5-1}=15$ *km/h* - speed of the second cyclist.

Answer: 12 *km/h;* 15 *km/h*.

№ 13.302 A circle of length *60 m* two points move evenly and in the same direction. One of them makes a full revolution *5 s* faster than the second. In this case, the coincidence of the points occurs after one minute..

Determine the speed of dots.

Solution:

Let the first point make one complete revolution in x seconds, and the second in y seconds.

Then $\frac{60}{x}\frac{м}{с}=\frac{60}{x}\frac{м}{\frac{1}{60}хв}=\frac{3600}{x}\frac{м}{хв}-$ the speed of the first point, and

$\frac{60}{y}\frac{м}{с}=\frac{60}{y}\frac{м}{\frac{1}{60}хв}=\frac{3600}{y}\frac{м}{хв}$. If $x<y,$ then we have the first equation $y-x=5\,(1)$

Since the points coincide every minute, the first point must go a full circle, that is, 60 m and another point that the second point goes through, that is

$\frac{3600}{x}=\frac{3600}{y}+60\,(2)$

We have such a system:

$\begin{cases}y-x=5,\\ \frac{3600}{x}=\frac{3600}{y}+60|:60\end{cases}\ \begin{cases}y=x+5,\\ \frac{60}{x}=\frac{60}{y}+1.\end{cases}$ We solve this system by the substitution method:

$\frac{60}{x}-\frac{60}{x+5}=1;\quad \frac{60x+300-60x}{x(x+5)}=1;\quad x(x+5)=300;\quad x^2+5x-300=0;\quad x_1=15;\quad x_2=-20.$

-20 - does not satisfy the condition of the problem.

$y_1=15+5=20.$

First point speed $\frac{60}{15}=4\left(\frac{м}{с}\right);$

Second point speed $\frac{60}{20}=3\left(\frac{м}{с}\right).$

Answer: $4\frac{м}{с};\ 3\frac{м}{с}.$

Collaboration tasks

№ 13.107 Two teams, working together, had to repair a certain part of the road in 18 days. But at first only the first brigade worked, and the second brigade itself finished the work, the labor productivity of which is higher than the first. The road repair lasted 40 days, and the first team completed $\frac{2}{3}$ of all work.

How many days would each team complete this work, working separately?

Solution:

Let 1 be all work;

the first team can complete it in x days, and the second in y days, then $\frac{1}{x}-$ productivity of the 1st brigade, $\frac{1}{y}-$ productivity of the 2nd brigade.

$\left(\frac{1}{x}+\frac{1}{y}\right)-$ joint labor productivity, $\left(\frac{1}{x}+\frac{1}{y}\right)\cdot 18-$ work performed by two teams in 18 days, that is, all work. We compose the first equation $\left(\frac{1}{x}+\frac{1}{y}\right)\cdot 18=1\,(1).$ Since the first brigade completed $\frac{2}{3}$ of all work, then she worked $\frac{2}{3}x$ дней. The second brigade completed $1-\frac{2}{3}=\frac{1}{3}$ all work, so she worked $\frac{1}{3}y$ days.

According to the problem statement, we compose the equation:

$\frac{2}{3}x+\frac{1}{3}y=40\,(2)$

We have a system of equations: $\begin{cases}\left(\frac{1}{x}+\frac{1}{y}\right)\cdot 18=1,\,(1)\\ \frac{2}{3}x+\frac{1}{3}y=40\,(2)\ \big|\cdot 3\end{cases}\quad \begin{cases}\left(\frac{1}{x}+\frac{1}{y}\right)\cdot 18=1,\\ 2x+y=120.\end{cases}$

$\begin{cases}\frac{x+y}{xy}\cdot 18=1\big|\cdot xy\\ y=120-2x.\end{cases}\quad \begin{cases}(x+y)\cdot 18=xy,\\ y=120-2x.\end{cases}\quad (x+120-2x)\cdot 18=x\cdot(120-2x);\quad (120-x)\cdot 18=120x-2x^2;$

$2x^2-120x+120\cdot18-18x=0;\ \rightarrow 2x^2-138x+2160=0 \,|:2;\ x^2-69x+1080=0;$

$D=4761-4320=441=21^2;\ x_1=\frac{69-21}{2}=\frac{48}{2}=24;\ x_2=\frac{69+21}{2}=45;\ y_1=120-2\cdot24=72;$

$y_2=30.$

Pairs of numbers $(24;72)$ and $(45;30)$ satisfy the system of equations.

However, a couple $(24;72)$ - does not satisfy the condition of the problem:

$\frac{1}{24}$ – labor productivity of the first brigade;

$\frac{1}{72}$ – labor productivity of the second brigade.

$\frac{1}{24}>\frac{1}{72}$, which contradicts the condition: labor productivity of the second brigade is higher than that of the first.

Another pair $(45;30)$ satisfies this condition $\frac{1}{45}<\frac{1}{30}$.

Answer: 45 days, 30 days.

№ 13.110 Two trains leave from points A and B towards each other. They will meet halfway if the train leaves point A 2 *hours* earlier, than a train with B. If both trains leave at the same time, then after 2 *ч* the distance between them will be $\frac{1}{4}$ distance between points A and B.

How long does each train take this distance??

Solution:

We will take the whole path for 1.

Let the first train cover the entire path in x *h*, and the second in y *h*.

The first will go half way in $\frac{x}{2}$ *h*, and the second for $\frac{y}{2}$ *h*.

Considering that the first train departed 2 *hours* earlier than the second, we compose the equation: $\frac{x}{2}-\frac{y}{2}=2$. $\frac{1}{x}\frac{км}{год}$ – first train speed, $\frac{1}{y}\frac{км}{год}$ – speed of the second.

The first train will pass in 2 *hours* $\frac{1}{x}\cdot2$ *(км)*. $\frac{1}{y}$ *км* - speed of the second.

In 2 *hours* it will pass $\frac{1}{y}\cdot2$ *(km)*. In case of simultaneous exit $1-\frac{1}{4}=\frac{3}{4}$ will be the distance between them. We have the second equation:

$\frac{2}{x}+\frac{2}{y}=\frac{3}{4}$. Let's solve the system of equations: $\begin{cases}\frac{x}{2}-\frac{y}{2}=2,\\ \frac{2}{x}+\frac{2}{y}=\frac{3}{4}.\end{cases}$

$\frac{x}{2}=2+\frac{y}{2}=\frac{4+y}{2};\ \frac{2}{x}=\frac{2}{4+y};\ \frac{2}{4+y}+\frac{2}{y}=\frac{3}{4};\ \frac{2y+8+2y}{y\cdot(4+y)}=\frac{3}{4};\ \frac{4y+8}{y\cdot(4+y)}=\frac{3}{4};\ \begin{matrix}y\neq0,\\ y\neq-4.\end{matrix}$

By the main property of the proportion, we have:

$4\cdot(4y+8)=3y\cdot(4+y)$; $16y+32=12y+3y^2$; $3y^2+12y-16y-32=0$; $3y^2-4y-32=0$; $D=16+384=400=20^2$.

$y_1=\frac{4-0}{6}=-\frac{16}{6}-$ does not satisfy the condition of the problem.

$y_2=\frac{4+20}{6}=\frac{24}{6}=4.$ $\frac{x}{2}-\frac{4}{2}=2$; $\frac{x}{2}=4$; $x=8.$

№ 13.290 If two pipes are open at the same time, the pool is filled up to *2 h 24 min*. In fact, at first, only the first pipe was opened during $\frac{1}{4}$ of the time, the required second pipe to fill the pool yourself. Then the second pipe also operated for $\frac{1}{4}$ of the time, necessary for the first pipe to fill the pool on its own. After that, it turned out that $\frac{11}{24}$ of the pool still needs to be filled.

How long does it take for each pipe to fill the pool on its own?

Solution:

2 *h* 24 *min* $=2\frac{24}{60}$ *h* $=2\frac{2}{5}$ *h*.

1 - the volume of the entire pool. Let the first pipe fill the pool in *x h*, and the second in *y h*, then the productivity of the first pipe will be $\frac{1}{x}$, and second $\frac{1}{y}$. At first the first pipe worked $\frac{1}{4}y$ *ч* and during this time filled $\frac{1}{x}\cdot\frac{y}{4}$ part of the pool, second pipe - $\frac{1}{y}\cdot\frac{x}{4}$ part of the pool, which, according to the condition of the problem, is $1-\frac{11}{24}=\frac{13}{24}$.

So, we have the equation $\frac{y}{4x}+\frac{x}{4y}=\frac{13}{24}$.

With simultaneous operation, the entire pool will be filled in $2\frac{2}{5}$ *h*, i.e

$$\left(\frac{1}{x}+\frac{1}{y}\right)\cdot 2\frac{2}{5}=1.$$

Let us solve the following system of equations:

$$\begin{cases}\frac{y}{4x}+\frac{x}{4y}=\frac{13}{24}\Big|\cdot 24\\ \left(\frac{1}{x}+\frac{1}{y}\right)\cdot 2\frac{2}{5}=1.\end{cases}\quad \begin{cases}6\cdot\frac{y}{x}+6\cdot\frac{x}{y}=13,\\ \frac{1}{x}+\frac{1}{y}=1:\frac{12}{5}.\end{cases}$$

We denote $\frac{y}{x}=t,\ t\neq 0.$ $\begin{cases}6\cdot t+\frac{6}{t}=13,\\ \frac{1}{x}+\frac{1}{y}=\frac{5}{12}\Big|\cdot 12xy\end{cases}$ $\begin{cases}6t^2-13t+6=0,\\ 12y+12x=5xy.\end{cases}$ $D=169-144=25=5^2$;

$$t_1=\frac{13-5}{12}=\frac{8}{12}=\frac{2}{3};\ t_2=\frac{13+5}{12}=\frac{18}{12}=\frac{3}{2}.$$

If $\frac{y}{x}=\frac{2}{3}$; $y=\frac{2}{3}x$; $12\cdot\frac{2}{3}x+12x=5x\cdot\frac{2}{3}x$; $20x=\frac{10}{3}x^2$; $\frac{10}{3}x^2-20x=0$; $10x\cdot\left(\frac{x}{3}-2\right)=0$,

$x\neq 0$, $\frac{x}{3}-2=0$, $x_1=6$. $y_1=\frac{2}{3}\cdot 6=4$. $(6;4)$

If $\frac{y}{x}=\frac{3}{2}$, то $y=\frac{3}{2}x$; $12\cdot\frac{3}{2}x+12x=5x\cdot\frac{3}{2}x$;

$18x+12x=\frac{15}{2}x^2$; $\frac{15x^2}{2}-30x=0$; $15x\cdot\left(\frac{x}{2}-x\right)=0$, $x\neq 0$, $\frac{x}{2}-2=0$; $x=4$. $y_2=\frac{3}{2}\cdot 4=6$

$(4;6)$

Answer: 4 *h*; 6 *h*.

Tasks of planning

№ 13.062 Turner apprentice grinds chess pawns for a certain number of chess sets. He wants to produce 2 pawns more daily than he does now, then he will complete the same task 10 days faster. If he produced 4 more pawns daily than now, then the period for completing the same task would decrease by 16 days.
How many sets of chess will this student provide with pawns if one set includes 16 pawns??

Solution:

Suppose it is required to grind out x pawns in total, for y pawns in 1 day, then he will complete the task in $\frac{x}{y}$ days. If in a day he will sharpen on $(y+2)$ pawns, then the task will be completed in $\frac{x}{y+2}$ days.

By the condition of the problem, we compose the equation: $\frac{x}{y}-\frac{x}{y+2}=10.$

If you produce on $(y+4)$ pawns, it will finish the job in $\frac{x}{y+4}$ days. Hence the equation: $\frac{x}{y}-\frac{x}{y+4}=16.$

Let us solve the following system of equations:

$$\begin{cases}\frac{x}{y}-\frac{x}{y+2}=10, \\ \frac{x}{y}-\frac{x}{y+4}=16.\end{cases}\left.\begin{matrix} y\neq 0, \\ y\neq -2, \\ y\neq -4.\end{matrix}\right\}-ОДЗН \quad \begin{cases}x(y+2)-xy=10y(y+2), \\ x(y+4)-xy=16y(y+4).\end{cases} \begin{cases}xy+2x-xy=10y^2+20y, \\ xy+4x-xy=16y^2+64y.\end{cases}$$

$$\begin{cases}2x=10y^2+20y, \mid\cdot(-2) \\ 4x=16y^2+64y.\end{cases}$$

$$+\begin{cases}-4x=-20y^2-40y, \\ 4x=16y^2+64y\end{cases}$$

$$0=-4y^2+4y.$$

$-4y^2+24y=0,$

$4y(-y+6)=0,$

$4y\neq 0,\ -y+6=0,\ y=6.$

$2x=10\cdot 36+20\cdot 6;\ 2x=480,\ x=\frac{480}{2}=240.$

We need to carve a total of 240 pawns.
Since 1 set of chess includes 16 pawns, they can provide $240:16=15$ комплектов.
Answer: 15 kits.

№ 13.328 A team of fishermen planned to catch 1800 centners of fish at a certain time. A third of this period was a storm, as a result of which the planned target was daily underfulfilled by 20 centners. However, on other days, the brigade caught 20 centners more than the daily norm daily, and therefore the planned task was completed 1 day before the deadline.
How many centners of fish the brigade had to catch daily?

Solution:

Suppose that in x days it was planned to catch 1,800 quintals of fish every day.
Then you can write the following equation $x\cdot y=1800$ (1). $\frac{1}{3}x$ days there was a storm. At this time, the brigade was fishing for $(y-20)$ centners daily. At this time, the brigade was catching $(y-20)\cdot\frac{1}{3}x$ centners.

$x-\frac{1}{3}x=\frac{2}{3}x$ days the weather was normal.

$\left(\frac{2}{3}x-1\right)$ days, the brigade actually worked after the storm, catching $(y-20)$ quintals of fish daily. During this period, she caught $(y+20)\cdot\left(\frac{2}{3}x-1\right)$ ц.

By the condition of the problem, we compose the equation:

$$(y-20)\cdot\frac{1}{3}x+(y+20)\cdot\left(\frac{2}{3}x-1\right)=1800\ (2).$$

Let us solve the following system of equations:

$$\begin{cases} x\cdot y=1800, (1) \\ (y-20)\cdot\frac{1}{3}x+(y+20)\cdot\left(\frac{2}{3}x-1\right)=1800\ (2). \end{cases}$$

$\frac{1}{3}xy-\frac{20x}{3}+\frac{2}{3}xy-y+\frac{40x}{3}-20=1800;$

$xy+\frac{20x}{3}-y-20=1800;$

$1800+\frac{20x}{3}-y-20=1800;$

$\frac{20x}{3}-y-20=0;$

$y=\frac{20x}{3}-20\ (3);$

Substituting (3) into (1):

$x\cdot\left(\frac{20x}{3}-20\right)=1800;\ \frac{20x}{3}-20x-1800=0\ |\cdot 3;$

$20x^2-60x-540=0\ |:20$

$x^2-3x-270=0.\ D=9+1080=1089=33^2.$

$x_1=\frac{3-33}{3}=-15-$ does not satisfy the condition of the problem.

$x_2=\frac{3+33}{3}=\frac{36}{2}=18.$

$y=\frac{1800}{18}=100.$

It was planned to catch 100 quintals of fish daily.

Answer: 100 c.

№ 13.177 According to the plan for a few months we had to produce 6,000 televisions. Having increased labor productivity by 70 TVs a month, already a month before the deadline, they exceeded the plan for 30 TVs.

How many months it was planned to make 6000 TVs?

Solution:

Suppose it was planned to make 6000 TVs in x months, then a month before the deadline, that is, for $(x-1)$ months produced $6000+30=6030$ *(TVs)*. According to the plan, they were supposed to produce monthly $\frac{6000}{x}$ TVs, but in reality they made $\frac{6030}{x-1}$ *(TVs)*.

According to the condition of the problem, we compose the equation:

$$\frac{6030}{x-1}-\frac{6000}{x}=70;$$

Range of valid values: $\left[\begin{array}{l}x-1\neq 0,\\ x\neq 0.\end{array}\right.\ \left[\begin{array}{l}x\neq 0,\\ x\neq 1.\end{array}\right.$

$6030x-6000x+6000=70x\cdot(x-1);$

$70x^2-70x-30x-6000=0;\ 70x^2-100x-6000=0\,|:10\ \ 7x^2-10x-600=0;$

$D=100+16800=16900=130^2;$

$x_1=\frac{10-130}{14}=-\frac{120}{14}-$ does not satisfy the condition of the problem;

$x_2=\frac{10+130}{14}=\frac{140}{14}=10.$

Answer: 10 months.

№ 13.180 The construction crew had to have $432\,м^3$ masonry. Since the brigade decreased by 4 people, each of them had to lay out on $9\,м^3$ more than planned. How many people were in the brigade first?

Solution:

First, there were x people in the brigade, and then it became $(x-1)$ person. Each member of the brigade must lay out $\frac{432}{x}\,м^3$ masonry, but actually laid out $\frac{432}{x-4}\,м^3$.

From the problem statement follows the equation $\frac{432}{x-4}-\frac{432}{x}=9 \big| :9$

Range of valid values: $\left[\begin{array}{l} x-4=0, \\ x\neq 0. \end{array}\right. \left[\begin{array}{l} x\neq 4, \\ x\neq 0. \end{array}\right.$

$\frac{48}{x-4}-\frac{48}{x}=1;\ 48x-48x+192=x\cdot(x-4);\ x^2-4x-192=0.$

By Vieta's theorem $x_1=16;\ x_2=-12-$ does not satisfy the condition of the problem.
Answer: 16 persons.

Tasks of the dependency between components

№ 13.040 At three box offices 1410 UAH. In the second $\frac{1}{3}$ the amount that is in the first and another 60 UAH. In the third $\frac{1}{3}$ the amount that is in the second and another 30 UAH.
How many hryvnias in each cash register?

Solution:

Let there be x UAH in the first box office, then in the second box it was $\left(\frac{1}{3}x+60\right)$ UAH, in the third $\left(\frac{1}{3}\cdot\left(\frac{1}{3}x+60\right)+30\right)$ UAH.

Together in the three box offices there were $\left(x+\frac{1}{3}x+60+\frac{1}{9}x+50\right)$ UAH, on condition that the task is 1410 UAH.

The equation: $x+\frac{1}{3}x+60+\frac{1}{9}x+50=1410;$

$\frac{9x+3x+x}{9}+110=1410;\ \frac{13x}{9}=1410-110;\ \frac{13x}{9}=1300;\ x=1300:\frac{13}{9}=\frac{1300}{1}\cdot\frac{9}{13}=900.$

UAH 900 was at the 1st box office.

$900\cdot\frac{1}{3}+60=360\ (грн)$ was at the 2st box office.

$360\cdot\frac{1}{3}+30=150\ (грн)$ was at the 3st box office.

Answer: 900 UAH; 360 UAH; 150 UAH.

№ 13.027 The sum of the squares of the digits of a two-digit number is 13. If you subtract 9 from this number, you get a number written in the same digits, but in reverse order. Find this number.

Solution:

Let the required number $\overline{xy}=10x+y,$ where x - tens digit, $x>0;$
y - number of units, $y>0.$
x^2+y^2- the sum of squares of digits, according to the condition of the problem is equal to 13.

$x^2+y^2=13.$ If on the desired number to subtract 9, that is, $\overline{xy}-9=\overline{yx}.$
$10x+y-9=10y+x.$ Let us solve the following system of equations:

$$\begin{cases} x^2+y^2=13, \\ 10x+y-9=10y+x \end{cases} \begin{cases} x^2+y^2=13, \\ 9x-9=9y \end{cases} \begin{cases} x^2+(x-1)^2=13, \\ y=x-1. \end{cases}$$

$x^2+x^2-2x+1-13=0,\ 2x^2-2x-12=0 \,|:2\ \ x^2-x-6=0,\ x_1=3,\ x_2=-2-$ does not satisfy the condition of the problem.

$y_1=3-1=2.$

Answer: 32.

№ 13.028 The numerators of the three fractions are proportional to the numbers 1, 2, 5, and the denominators, respectively, are proportional to the numbers 1, 3, 7. The arithmetic mean of these fractions is $\frac{200}{441}$.

Find these fractions.

Solution:

Let be *x* - the proportionality coefficient of the numerators of the fraction, then they are equal *x*, *2x, 5x.*

y - the proportionality coefficient of the denominators, then they are equal *y, 3y, 7y.*

Then the given fractions will have the form: $\frac{x}{y};\ \frac{2x}{3y};\ \frac{5x}{7y}.$

Their arithmetic mean is: $\frac{\frac{x}{y}+\frac{2x}{3y}+\frac{5x}{7y}}{3}$, which, according to the problem statement, is $\frac{200}{441}$.

We have the equation $\frac{\frac{x}{y}+\frac{2x}{3y}+\frac{5x}{7y}}{3}=\frac{200}{441};\ \frac{x}{y}\cdot\frac{21+14+15}{21}=\frac{200}{441}\cdot 3;\ \frac{x}{y}\cdot\frac{50}{21}=\frac{200}{147};$

$\frac{x}{y}=\frac{200}{147}\cdot\frac{21}{50}=\frac{4}{7};$

First fraction $\frac{x}{y}=\frac{4}{7};$ Second fraction $\frac{2x}{3y}=\frac{2\cdot4}{3\cdot7}=\frac{8}{21};$ Third fraction $\frac{5x}{7y}=\frac{5\cdot4}{7\cdot7}=\frac{20}{49}.$

Answer: $\frac{4}{7};\ \frac{8}{21};\ \frac{20}{49}.$

№ 13.061 We bought 4 different brands for the amount of 2 UAH 80 kopecks. The values of these marks make up an arithmetic progression, in which the largest term is 2.5 times larger than the smallest of them.

How much does each mark?

Solution:

Let *x* UAH. the cheapest brand costs, then the most expensive brand costs 2,5 UAH. Find the sum of four terms ÷ according to the formula $S_n=\frac{a_1+a_n}{2}\cdot n;$

$S_4 = \frac{x+2,5x}{2}\cdot 4 = 3,5x\cdot 27x;$ By condition $S_4 = 2,8$ UAH. Hence we have the equation $7x = 2,8;\ \ x = 2,8 : 7 = 0,4.$

$a_1 = 0,4;\ \ a_4 = 2,5\cdot 0,4 = 1;$ Using the formula for the nth term of the arithmetic progression, we find its difference d.

$a_n = a_1 + d(n-1);\ \ a_4 = a_1 + d\cdot(4-1);\ \ 0,4 + 3d = 1;\ \ 3d = 0,6;\ \ d = 0,2.$

The first stamp is worth 0,4 UAH

The second mark is worth $0,4 + 0,2 = 0,6$ UAH

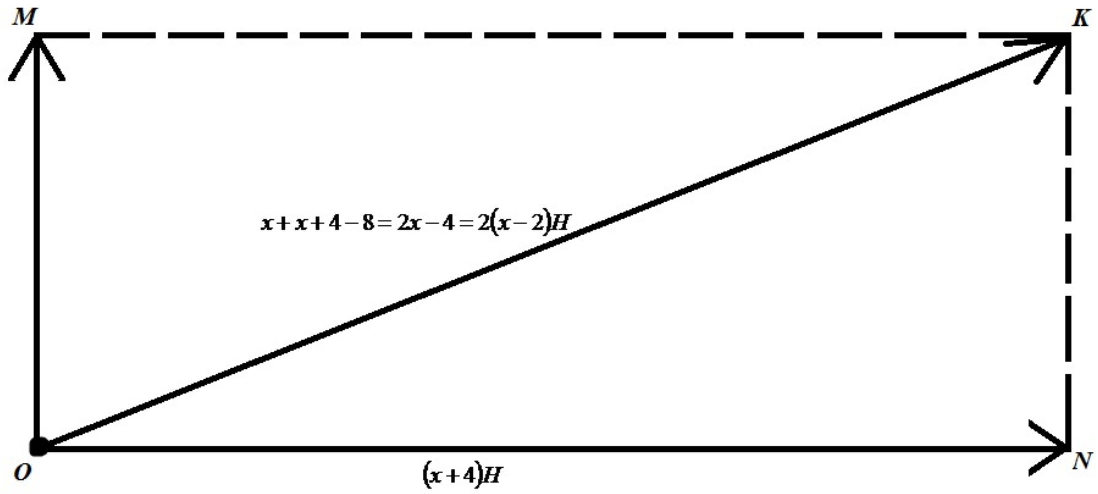
The third mark is worth $0,6 + 0,2 = 0,8$ UAH

The fourth mark is worth 1,2 UAH

№ 13.057 Two forces applied at one point are mutually perpendicular. The value of one of them is 4N more than the value of the second, and the value of the resultant is 8N less than the sum of the values of these forces.
Find the magnitudes of these forces and their resultant.

Solution:

Let the magnitude of one force be x_H, then another - $(x+4)H$.
The resultant of these forces is the diagonal of the rectangle built on these sides.



In the $\Delta ONK\ (\angle N = 90^\circ)$ by the Pythagorean theorem, we have:

$OK^2 = KN^2 + ON^2,\ \ (2(x+2))^2 = x^2 + (x+4)^2,\ \ 4(x^2 - 4x + 4) = x^2 + x^2 + 8x + 16,$

$4x^2 - 16x + 16 - 2x^2 - 8x - 16 = 0,\ \ 2x^2 - 24x = 0,\ \ 2x(x-12) = 0,\ \ \left[\begin{array}{l} 2x = 0, \\ x - 12 = 0. \end{array}\right.$

$\left[\begin{array}{l} x = 0; \text{–не задовольняє умову заддачі.} \\ x = 12 \end{array}\right.$

The magnitude of the force directed along the vector *OM* is equal to 12*N*, along the vector *ON* is equal to $12 + 4 = 16H$, resultant $2\cdot(12-2) = 20H$.

Answer: *12N, 16N, 20N.*

<u>Tasks on a percents</u>

<u>Tip: replace percentages with fractions (preferably decimal)</u>.

№ 13.007 The price of the product was first reduced by 20%, then the new price was reduced by 15%, and after a while - by 10%.

By what percentage have the original price of the product been reduced?

Solution:

Let the initial price of the product x UAH, which is 100%. $20\%=0{,}2;\ 15\%=0{,}15;$ $10\%=0{,}1.$ Then $0{,}2x$ UAH is the first reduction of the product, and its price will be $x-0{,}2x=0{,}8x$ UAH.

The second decline is $0{,}15\cdot 0{,}8x=0{,}12x$ UAH, and the price after the second decline is $0{,}8x-0{,}12x=0{,}68x$ UAH.

Third price cut $0{,}68x\cdot 0{,}1=0{,}068x$ UAH.

Price after the third decline $0{,}68x-0{,}068x=0{,}612x$ UAH. For three times the price of the product has decreased by $x-0{,}612x=0{,}388x$ UAH.

We make the proportion: let be $t\%$ is $0{,}388$ UAH.

$x-100\%$
$0{,}388x-t\%$ $\frac{x}{0{,}388x}=\frac{100}{t};\ \frac{1}{0{,}388}=\frac{100}{t};$ By the main property of the proportion, we have: $t\cdot 1=0{,}388\cdot 100;\ t=38{,}8.$

The initial price of the product has decreased by $38{,}8\%.$

Answer: on $38{,}8\%.$

№ 13.049 One of the unknown numbers is 140% of the second, and the ratio of the first and third is $\frac{14}{11}$. The difference between the third and the second is 40 units less than the number, which is 12.5% of the sum of the first and second numbers. Find these numbers.

Solution:

$140\%=1{,}4;\ 12{,}5\%=0{,}125.$

Let be x - second number then $1{,}4x-$ first number.

By the condition of the task $\frac{1{,}4x}{III\ \text{число}}=\frac{14}{11};$

a third number $=1{,}4x:\frac{14}{11}=\frac{1{,}4x}{1}\cdot\frac{11}{14}=\frac{11x}{10}=1{,}1x.$

$1{,}1x-x=0{,}1x-$ difference between third and second number.

$0{,}125\cdot(1{,}4x+x)=0{,}125\cdot 2{,}4x=0{,}3x$ is equal $12{,}5\%$ sums of the first and second numbers.

According to the condition of the problem, we compose the equation:

$0{,}1x=0{,}3x-40;\ -0{,}2x=-40;$

$x=-40:(-0{,}2)=200.\ 200-$ the second number.

$200\cdot 1{,}4=280-$ the first number,

$200\cdot 1{,}1=220-$ a third number.

Answer: 280; 200; 220.

№ 13.154 For 1 kg of one product and 10 kg of the second, they paid 2 UAH. If the first product rises in price by 15%, and the second falls in price by 25%, then for the same amount of these products you have to pay 1 UAH 82 kopecks.

How much does 1 kg of each product cost??

Solution:

Let be *x* UAH. costs 1 kg of the first product, *y* UAH. - 1 kg of the second product.
$15\% = 0{,}15;\ 25\% = 0{,}25;$ 1 UAH 82 kopecks. = 1,82 UAH.
After the price of the first product has risen, 1 kg will cost $(x + 0{,}15x)$.
After the price reduction, 1 kg of the second product will cost $(y - 0{,}25y)$.
According to the conditions of the problem, we compose the system of equations:
$$\begin{cases} x + 10y = 2, \\ x + 0{,}15x + 10(y - 0{,}25y) = 1{,}82. \end{cases} \quad \begin{cases} x = 2 - 10y, \\ 1{,}15x + 7{,}5y = 1{,}82. \end{cases}$$
$1{,}15 \cdot (2 - 10y) + 7{,}5y = 1{,}82;\ 2{,}3 - 11{,}5y + 7{,}5y = 1{,}82;\ -4y = -0{,}48;\ y = -0{,}48 : (-4) = 0{,}12.$
$x = 2 - 10 \cdot 0{,}12 = 2 - 1{,}2 = 0{,}8.$
0,8 грн. costs 1 kg of the first product.
0,12 грн. costs 1 kg of the second product.
Answer: *0,8 UAH; 0,12 UAH.*

Tasks for mixtures and alloys

№ 13.041 Mixed 30% hydrochloric acid solution with 10% and received 600 g of 15% solution.
How many grams of each solution were taken?

Solution:

$30\% = 0{,}3;\ 10\% = 0{,}1;\ 15\% = 0{,}15.$
Let the 30% solution take *x* g, and 10% - *y* g. Then hydrochloric acid in a 30% solution contains 0,3*x* g, and in 10% - 0,14*y* g.
In a 15% solution $0{,}15 \cdot 600 = 90$ (g) acid.
According to the conditions of the problem, we compose the following system of equations:
$$\begin{cases} x + y = 600\ (1) \\ 0{,}3x + 0{,}1y = 90\ (2) \,|\cdot(-10) \end{cases} \quad + \begin{cases} x + y = 600 \\ -3x - y = -900 \end{cases}$$
$$-2x = -300;$$
$$x = \frac{-300}{-2} = 150.$$
$150 + y = 600;\ y = 600 - 150 = 450.$
Answer: *150 g; 450 g.*
Задача. Part of the alcohol was poured from a vessel filled with alcohol and the same amount of water was added. Then, as many liters of the mixture were poured from the vessel as the alcohol was poured for the first time, after which 49 liters of pure alcohol remained in the vessel. Filling the vessel contained 64 liters of alcohol.
How many liters of alcohol were poured from the vessel each time?

Solution:

Let *x* (*l*) alcohol be poured for the first time, then $(64 - x)$ *l* alcohol left.
After *x* (*l*)water was added to the vessel, the mixture in it became $64 - x + x = 64$ *(l)*.

One liter of the mixture contains $\frac{64-x}{64}$ *l* alcohol. The second time, *x* mixtures were cast, they contain $\frac{64-x}{64}\cdot x$ *l* alcohol.
Twice cast from the vessel $64-49=15$ *(l)* alcohol. On the other hand, the alcohol cast is $\left(x+\frac{64-x}{64}\cdot x\right)$ *l*. We have the equation:

$$x+\frac{64-x}{64}\cdot x=15\,\Big|\cdot 64 \quad 64x+(64-x)\cdot x=15\cdot 64; \quad 64x+64x-x^2=960; \quad x^2-128x+960=0;$$

$$D=16384-3840=12544=112^2; \quad x_1=\frac{128-112}{2}=8; \quad x_2=\frac{128+112}{2}=\frac{240}{2}=120.$$

120 *l* - does not satisfy the condition of the problem because $120>64$.
8 *l* alcohol was cast for the first time.
$\frac{64-8}{64}\cdot 8=\frac{56}{8}=7$ *(l)* alcohol was cast a second time.
Answer: *8 l; 7 l.*

Self-study assignments:

Problems of the collection of competitive problems edited by M.I. Skanavi
№ 13.077 Older brother on a motorcycle, and the youngest on the bike made a two-hour trip without stopping in the woods and back. At the same time, a motorcyclist traveled every kilometer 4 minutes faster than a cyclist.
How many kilometers did each of the brothers traveled in 2 hours, if it is known that the older brother traveled 40 km longer during this time?.
Answer: *20 km; 60 km.*
№ 13.222 The distance between stations A and B is 103 km. From A to B, a train came out, which, after a certain stop, continued to move at a speed of 4km/h more from the previous one.
Find the initial speed of the train if the remaining path to B is 23 km longer than the path traveled to the stop, and the path after the stop is 15 minutes longer than before the stop.
Answer: 80 km/h
№ 13.112 Two bodies move towards each other from two points, the distance between which is 390 m. Первое the body passed 6 m in the first second, and the second for each next one passed 6 m more than the previous one. The second body moved uniformly at a speed of 12 m/s and began to move 5 s later than the first. Через сколько секунд после начала движения первого тела они встретятся?
Answer: after 10 s.
№ 13.130 Two comrades rode in the same boat along the river bank downstream and returned along the same river route 5 hours after the start of the movement. The entire voyage is 10 km. According to their calculations, it turned out that for every 2 km upstream, it takes as much time as 3 km downstream.

Find the current speed and time of movement along the river flow and time upstream.
Answer: $\frac{5}{12}$ km/h; 2 h; 3 h.
№ 13.284 Find the speed and length of a train if it passes at a constant speed past a stationary observer for 7 s and spent 25 s to travel at the same speed along a platform 378 m long.
Answer: 75,6 km/h and 147 m.
№ 13.317 One tourist left at 6 o'clock, and the second - towards him at 7 o'clock. They met at 8 o'clock and without stopping, continued to move.
How much time did each of them spend on the entire journey, if the first went to the place from which the second left, 28 minutes later than the second came to the place from which the first left?
It is believed that everyone moved non-stop at a constant speed..
Answer: 3 h 10 min; 2 h 12 min.
№ 13.096 A cyclist travels 500 m less per minute than a motorcyclist, and therefore spends 2 hours more time on a path of 120 km than a motorcyclist. Calculate the speed of each of them.
Answer: 30 km/h; 60 km/h.
№ 13.126 Two points move evenly along two concentric circles. One of them makes a full revolution 5 s faster than the second, and therefore manages to make 2 more revolutions in 1 minute..
How many revolutions per minute does each point?
Answer: 4; 6.
№ 13.135 A team of locksmiths can complete a specific task 15 hours faster than a team of apprentices. If a team of students spends 18 hours, and then a team of locksmiths continues to complete this task for 6 hours, then only 2/3 of the entire task will be completed..
How long does it take for a team of students to complete this task on their own?
Answer: 45 h.
№ 13.142 The vessel is filled with two taps A and B. Filling the vessel only through tap A takes 22 minutes longer than through tap B. If you open both taps, the vessel will be filled in 1 hour.
Answer: 2 h 12 min; 1 h 50 min.
Задача The turner must produce 272 parts in a specified time. 10 days after the start of work, the turner began to overfulfill the daily norm by 4 parts and therefore, the day before the deadline, he produced 280 parts.
How many parts will the turner make on time?
Answer: 300 parts.
№ 13.055 At the railway carriage repair plant, 330 railway carriage were to be repaired within a certain period of time. Repairing 3 more carriages in a week, 297 carriages were repaired two weeks before the deadline.
How many railway carriage were repaired per week?
Answer: 33 railway carriage.

Задача The team had to produce 8000 parts in a certain time. In fact, she finished work 8 days early because she was producing 50 more parts daily than planned.. How many days it was planned to produce 8000 parts?
Answer: 40 days.
№ 13.181 A team of workers in the electric lamp shop had to make 7200 parts per shift, and each worker had to make the same number of lamps. Three workers worked slowly, so each worker had to make 400 more parts to complete the plan.. How many workers were in the brigade?
Answer: 9 workers.
№ 13.015 The tourist traveled the distance between cities in 3 days. On the first day he drove $\frac{1}{5}$ all the way and another 60 km, for the second - $\frac{1}{4}$ all the way and another 20 km, for the third day $\frac{23}{80}$ all the way and 25 km that is left.
Find out the distance between cities?
Answer: 400 km.
№ 13.119 The product of the digits of a two-digit number is three times less than the number itself. If 18 is added to the desired number, then a number is formed written in the same digits, but in reverse order.
Find this number.
Answer: 24.
№ 13.042 The areas of the three plots of land are proportional to the numbers $\frac{11}{14}$; $\frac{11}{6}$; і $\frac{11}{8}$.
From the first section, grain was harvested by 72 centners more than from the other. The average yield is 18 centners per hectare.
Find the area of all three parcels.
Answer: 26 hectare.
Задача Find four numbers that form a proportion if the sum of the extreme terms is 14, the sum of the middle terms is 11, and the sum of the squares of these four numbers is 221.
Answer: 12; 8; 3; 2.
№ 13.344 The magnitudes of two forces acting on a material point at a right angle, and the magnitude of their resultant form an arithmetic progression.
What is the relation of magnitude of these numbers are?
Answer: 3:4:5.
№ 13.013 In January, the plant fulfilled 105% of the monthly production plan. In February, he gave 4% more than in January.
How many percent did the plant exceed the two-month production plan??
Answer: by 7,1%.
№ 13.036 Fresh mushrooms contain 90% water by weight, and dry ones - 12% water.
How many dry mushrooms are formed from 22 kg of fresh?
Answer: 2,5 kg.

№ 13.075 Two workers produced 72 parts together per shift. After the first one increased its labor productivity by 15%, and the second one - by 25%, together they began to produce 86 parts per shift.
How many parts per shift each worker produces after increasing labor productivity?
Answer: 46 details; 40 details.
№ 13.090 There is a piece of copper-tin alloy with a total weight of 12 kg, containing 45% copper. How much pure tin needs to be added to this piece so that the new alloy contains 40% copper?
Answer: 1,5 kg.
№ 13.341 Several liters were poured from a vessel filled with acid and poured with water, then the same number of liters of solution was poured again, then 24 liters of pure acid remained in the vessel..
Объем сосуда 54 *l*.
How many liters of acid were poured for the first time and how many for the second time?
Answer: 18 *l*; 12 *l*.
Задача The worker received UAH 300 salary per month. Salary increased by 10%, and the cost of goods increased by 5%.
By what percentage has the real wage of a worker increased?
Answer: by 5%.
Задача The mined coal contains 2% water. After a while, the mass fraction of water is 15%.
How much has the mass of 17 tons of mined coal increased during the same time??
Answer: by 2,21 t.
Задача For the installation of the lowest reinforced concrete ring of the well, they paid 26 UAH, and for each subsequent ring they paid 2 UAH less than the previous one. In addition, at the end of the work they paid another 40 hryvnyas. The average cost of the installed ring turned out to be $22\frac{4}{9}$ UAH.
How many rings were installed?
Answer: 9 rings.