

## Section 2

# Converting algebraic expressions

Algebraic expressions are those expressions in which the actions of addition, subtraction, multiplication, division, raising to a rational power and extraction of a root are performed on the numbers and variables included in them.

For example:  $\frac{6c-d}{7e^2+4}$ ;  $n^{\frac{4}{5}} \cdot m^{0,75} - m^{\frac{1}{2}}$ ;  $\frac{\sqrt{x+y}}{x^2-y^2}$ .

Rational are algebraic expressions in which only the actions of addition, subtraction, multiplication, division and exponentiation with a natural exponent are performed.

For example:  $\frac{(a^{-3} - e^{-3}) \cdot (a^4 + e^4)}{\sqrt{3a^2 - 5a + (a - e)^2}}$ .

A rational expression in which there is no variable in the denominator is called a whole rational expression.

For example:  $(m^2 - n^2) \cdot (3m^5 + 4n^6)$ .

A fractional rational expression is an expression containing a variable in the denominator of the fraction.

For example:  $(x^{-3} + y^{-3}) \cdot \frac{(x^2 + y^2) \cdot (x^2 - y^2)}{2xy}$ .

Irrational are algebraic expressions in which the actions of raising to a power with a fractional exponent, or extracting a root.

For example:  $(x + y + z)^{\frac{2}{3}}$ ;  $\sqrt{4 + \sqrt{2 - x}}$ .

Expression conversion has the goal of simplifying them. This is achieved in the following ways:

- a). adding similar members to polynomials;
- б). converting the numerator and denominator into products, taking the common factor out of parentheses and reducing fractions;
- в). applying abbreviated multiplication formulas:

$$(a+e)^2 = a^2 + 2ae + e^2;$$

$$(a - e)^2 = a^2 - 2ae + e^2;$$

$$a^2 - e^2 = (a - e) \cdot (a + e);$$

$$(a+e)^3 = a^3 + 3a^2e + 3ae^2 + e^3;$$

$$(a - e)^3 = a^3 - 3a^2e + 3ae^2 - e^3;$$

$$a^3 + e^3 = (a+e) \cdot (a^2 - ae + e^2);$$

$$a^3 - e^3 = (a - e) \cdot (a^2 + ae + e^2).$$

$ax^2 + bx + c$  – square trinomial,

$x_1, x_2$  – square trinomial roots,

$ax^2 + bx + c = a \cdot (x - x_1) \cdot (x - x_2)$  – the formula for the factorization of a square trinomial.

## "Tools" when converting algebraic expressions

there are such formulas:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad (b \cdot d \neq 0);$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \quad (bd \neq 0);$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad (bd \neq 0);$$

$$\frac{a}{b} : \frac{c}{d} = \frac{a \cdot d}{b \cdot c} \quad (bcd \neq 0).$$

Keep in mind these four tips when you start directly converting expressions:

- 1). Do not divide by 0;
- 2). Do not extract an even root of a negative number;
- 3). Don't look for the logarithms of negative numbers with a negative base and a base equal to 1;
- 4). Remember, that  $|\sin x| \leq 1$  и  $|\cos x| \leq 1$ .

It is useful to recall the algorithm for raising several fractions to a common denominator:

1. find L.C.D. (*lowest common denominator*) these fractions;
2. divide found L.C.D. by the denominator of each fraction and find additional factors for each fraction;
3. multiply the numerator of each fraction by the corresponding additional factor and write these products in the numerator, and the found L.C.D. - in the denominator.

When raising algebraic fractions to a common denominator, it is advisable to clearly differentiate each stage of work according to such a scheme:

Fraction denominator	L.C.D. fractions	Additional multipliers
$x$	$x \cdot (x - 2) \cdot (x + 2)$	$x^2 - 4$
$x - 2$		$x^2 + 2x$
$x + 2$		$x^2 - 2x$
1	$(x - 3) \cdot (x^2 + 3x + 9)$	$x^3 - 27$
$x - 3$		$x^2 + 3x + 9$
$x^2 + 3x + 9$		$x - 3$

The introduction to the workshop in this section could be the following exercises:

**2.1** At what value of the parameter  $a$  square trinomial  $25x^2 + 30x + a$  can be written as a complete square of the sum of two monomials.

Solution:

We transform the given trinomial  $25x^2 + 30x + a = 5^2 \cdot x^2 + 2 \cdot 5 \cdot x \cdot 3 + a = (5 \cdot x)^2 + 2(5 \cdot x) \cdot 3 + a = (5 \cdot x)^2 + 2 \cdot 5x \cdot 3 + 3^2 =$

$$=(5x)^2+30x+9.$$

Comparing the beginning of this phrase with the end, you can see that  $a=9$ .

$(5 \cdot x + 3^2)$ – is the square of the sum of two monomials.

In such exercises, direct substitution of the variable value into an expression leads to cumbersome calculations, you should first simplify the expressions.

For example, calculate the value of the expression:

$$2.2 \frac{\sqrt[3]{25} \cdot \epsilon^{\frac{2}{3}} - 4}{\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} + 2} - \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}}. \text{ at } \epsilon=0,0025.$$

Solution:

$$\begin{aligned} \frac{\sqrt[3]{25} \cdot \epsilon^{\frac{2}{3}} - 4}{\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} + 2} - \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} &= \frac{\sqrt[3]{5^2} \cdot \left(\epsilon^{\frac{1}{3}}\right)^2 - 2^2}{\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} + 2} - \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} = \frac{\left(\sqrt[3]{5}\right)^2 \cdot \left(\epsilon^{\frac{1}{3}}\right)^2 - 2^2}{\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} + 2} - \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} = \\ &= \frac{\left(\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}}\right)^2 - 2^2}{\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} + 2} - \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} = \frac{\left(\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} - 2\right) \cdot \left(\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} + 2\right)}{\sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} + 2} - \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} = \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} - 2 - \sqrt[3]{5} \cdot \epsilon^{\frac{1}{3}} = -2. \end{aligned}$$

Answer:  $-2$ .

$$2.3 \text{ Calculate: } \frac{\sqrt{x} + 2}{x\sqrt{x} + 2x + 4\sqrt{x}} : \frac{1}{x^2 - 8\sqrt{x}} \text{ at } x=4,1.$$

Solution:

At first, you need to simplify this expression by replacing the roots with powers

with fractional exponents:  $\frac{\sqrt{x} + 2}{x\sqrt{x} + 2x + 4\sqrt{x}} : \frac{1}{x^2 - 8\sqrt{x}} =$

$$\begin{aligned} \frac{x^{\frac{1}{2}} + 2}{x^{\frac{3}{2}} + 2x + 4 \cdot x^{\frac{1}{2}}} \cdot \frac{1}{x^2 - 8 \cdot x^{\frac{1}{2}}} &= \frac{\left(x^{\frac{1}{2}} + 2\right) \cdot x^{\frac{1}{2}} \cdot \left(x^{\frac{3}{2}} - 8\right)}{x^2 \left(x + 2x^{\frac{1}{2}} + 4\right)} = \frac{\left(x^{\frac{1}{2}} + 2\right) \left(\left(x^{\frac{1}{2}}\right)^3 - 2^3\right)}{x + 2x^{\frac{1}{2}} + 4} = \\ &= \frac{\left(x^{\frac{1}{2}} + 2\right) \cdot \left(x^{\frac{1}{2}} - 2\right) \cdot \left(x + 2x^{\frac{1}{2}} + 4\right)}{x + 2x^{\frac{1}{2}} + 4} = x - 4. \end{aligned}$$

If  $x=4,1$ , then  $x - 4 = 4,1 - 4 = 0,1$ .

Answer:  $0,1$ .

Calculate:

$$2.4 \frac{\sqrt{x} + 7}{x\sqrt{x} + 7x + 49\sqrt{x}} : \frac{1}{x^2 - 343\sqrt{x}}, \text{ if } x=9,1.$$

Solution:

$$\frac{\sqrt{x}+7}{x\sqrt{x}+7x+49\sqrt{x}} \cdot \frac{1}{x^2-343\sqrt{x}} = \frac{x^{\frac{1}{2}}+7}{x^{\frac{1}{2}}\left(x+7x^{\frac{1}{2}}+49\right)} \cdot \frac{x^{\frac{1}{2}}\left(x^{\frac{3}{2}}-7^3\right)}{1} =$$

$$= \frac{\left(x^{\frac{1}{2}}+7\right) \cdot x^{\frac{1}{2}} \cdot \left(x^{\frac{1}{2}}-7\right)\left(x+7x^{\frac{1}{2}}+49\right)}{x^{\frac{1}{2}}\left(x+7x^{\frac{1}{2}}+49\right)} = x-49.$$

If  $x=9,1$ , then  $x-49=9,1-49=-39,9$ .

Answer:  $-39,9$ .

Simplify expression: **2.5**  $\frac{a}{a^2-1} + \frac{a^2+a-1}{a^3-a^2+a-1} + \frac{a^2-a-1}{a^3+a^2+a+1} - \frac{3a^3}{a^4-1}$ .

Solution:

R.O.V.V (*range of valid values*):  $a \neq \pm 1$ . Using the abbreviated multiplication formulas, we factor out the denominator of each fraction:

$$a^2-1 = a^2 - 1^2 = (a-1) \cdot (a+1);$$

$$a^3 - a^2 + a - 1 = (a^3 - a^2) + (a - 1) = a^2(a-1) + 1 \cdot (a-1) = (a-1) \cdot (a^2 + 1);$$

$$a^3 + a^2 + a + 1 = a^2(a+1) + (a+1) = (a+1) \cdot (a^2 + 1);$$

$$a^4 - 1 = a^4 - 1^4 = (a^2 - 1^2)(a^2 + 1^2) = (a-1) \cdot (a+1)(a^2 + 1).$$

$$\text{LCD (lowest common denominator)} = (a-1)(a+1)(a^2 + 1).$$

Divide LCD by each of the four denominators of these fractions.

We obtain additional factors for:

$$\text{first fraction } \frac{(a-1)(a+1)(a^2+1)}{a^2-1} = a^2 + 1;$$

$$\text{second fraction } \frac{(a-1)(a+1)(a^2+1)}{(a-1)(a^2+1)} = a + 1;$$

$$\text{third fraction } \frac{(a-1)(a+1)(a^2+1)}{(a+1)(a^2+1)} = a - 1;$$

$$\text{fourth fraction } \frac{(a-1)(a+1)(a^2+1)}{(a-1)(a+1)(a^2+1)} = 1;$$

In this way,

$$\frac{a}{a^2-1} + \frac{a^2+a-1}{a^3-a^2+a-1} + \frac{a^2-a-1}{a^3+a^2+a+1} - \frac{3a^3}{a^4-1} =$$

$$= \frac{a(a^2+1) + (a^2+a-1) \cdot (a+1) + (a^2-a-1) \cdot (a-1) - 3a^3 \cdot 1}{(a-1)(a+1)(a^2+1)} =$$

$$= \frac{a^3 + a + a^3 + a^2 + a^2 + a - a - 1 + a^3 - a^2 - a^2 + a - a + 1 - 3a^3}{(a-1)(a+1)(a^2+1)} = \frac{3a^3 + a - 3a^3}{a^4-1} = \frac{a}{a^4-1}.$$

Answer:  $\frac{a}{a^4-1}$  at  $a \neq \pm 1$ .

Solution:

$$\frac{(x^2 + 5x + 6) \cdot (x + 1)}{(x^2 - 1)(x + 3)} = \frac{(x + 2)(x + 3)(x + 1)}{(x - 1)(x + 1)(x + 3)} = \frac{x + 2}{x - 1}$$

$$\text{ODЗ: } \begin{cases} x \neq -1, \\ x \neq 1, \\ x \neq -3. \end{cases}$$

$x^2 + 5x + 6 = 0$ . By the theorem of Vieta:

$x_1 = -2$ ;  $x_2 = -3$ . Тогда

$$x^2 - 5x + 6 = (x + 2) \cdot (x + 3).$$

Answer:  $\frac{x + 2}{x - 1}$  at  $\begin{cases} x \neq \pm 1, \\ x \neq -3. \end{cases}$

Simplify expression:

$$2.6 \quad \frac{a^2 - a\epsilon + \epsilon^2}{x^2 - y^2} : \frac{a^3 + \epsilon^3}{x^2 - 2xy + y^2} = \frac{a^2 - a\epsilon + \epsilon^2}{(x - y)(x + y)} \cdot \frac{(x - y)^2}{(a + \epsilon) \cdot (a^2 - a\epsilon + \epsilon^2)} =$$

$$= \frac{(a^2 - a\epsilon + \epsilon^2) \cdot (x - y)^2}{(x - y) \cdot (x + y) \cdot (a + \epsilon) \cdot (a^2 - a\epsilon + \epsilon^2)} = \frac{x - y}{(x + y) \cdot (a + \epsilon)}$$

Answer:  $\frac{x - y}{(x + y)(a + \epsilon)}$  at  $\begin{cases} x \neq \pm y, \\ a \neq -\epsilon. \end{cases}$

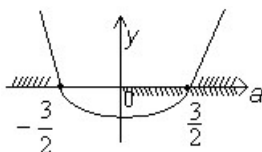
2.7 Simplify expression:  $\sqrt{4a - 2} \cdot \sqrt{4a^2 - 9}$ .

Solution:

Since this expression does not exist for all values of the variable  $a$ , we find R.O.V.V (range of valid values):

$$\begin{cases} 4a^2 - 9 \geq 0, \\ 4a - 2\sqrt{4a^2 - 9} \geq 0. \end{cases} \quad \begin{cases} 4a^2 - 9 \geq 0, \\ 4a \geq 2 \cdot \sqrt{4a^2 - 9}. \end{cases} \quad \begin{cases} (2a - 3) \cdot (2a + 3) \geq 0, \\ 16a^2 \geq 4 \cdot (4a^2 - 9), \\ a \geq 0. \end{cases}$$

$$\begin{cases} \left(a - \frac{3}{2}\right) \cdot \left(a + \frac{3}{2}\right) \geq 0 \\ 16a^2 \geq 16a^2 - 36 \\ a \geq 0. \end{cases}$$



$a \in \left[\frac{3}{2}; +\infty\right)$  Well then, R.O.V.V of this expression  $\left[\frac{3}{2}; +\infty\right)$ .

In the range of valid values, let's transform this expression.

You can convert the expression first:

$$4a - 2\sqrt{4a^2 - 9} = (2a - 3) + (2a + 3) - 2\sqrt{(2a - 3) \cdot (2a + 3)} = (\sqrt{2a - 3})^2 - 2\sqrt{(2a - 3) \cdot (2a + 3)} + (\sqrt{2a + 3})^2 = (\sqrt{2a - 3} - \sqrt{2a + 3})^2.$$

Then  $\sqrt{4a - 2\sqrt{4a^2 - 9}} = \sqrt{(\sqrt{2a - 3} - \sqrt{2a + 3})^2} = |\sqrt{2a - 3} - \sqrt{2a + 3}|$ .

Considering that  $a \geq \frac{3}{2}$  we have  $|\sqrt{2a - 3} - \sqrt{2a + 3}| = -$

$$(\sqrt{2a - 3} - \sqrt{2a + 3}) = -\sqrt{2a - 3} + \sqrt{2a + 3} = \sqrt{2a + 3} - \sqrt{2a - 3}.$$

Answer:  $\sqrt{2a - 3} - \sqrt{2a + 3}$ .

Simplify the expression:

**2.8**  $\left(\frac{1}{x - y} + \frac{3xy}{y^3 - x^3}\right) : \left(\frac{x^2 + y^2}{x^2 - y^2} - \frac{x + y}{2x - 2y}\right)$ .

Solution:

$$\begin{aligned} & \left(\frac{1}{x - y} + \frac{3xy}{y^3 - x^3}\right) : \left(\frac{x^2 + y^2}{x^2 - y^2} - \frac{x + y}{2x - 2y}\right) = \left(\frac{1}{x - y} - \frac{3xy}{y^3 - x^3}\right) : \left(\frac{x^2 + y^2}{x^2 - y^2} - \frac{x + y}{2x - 2y}\right) = \\ & = \left(\frac{1}{x - y} - \frac{3xy}{(x - y) \cdot (x^2 + xy + y^2)}\right) : \left(\frac{x^2 + y^2}{(x - y) \cdot (x + y)} - \frac{x + y}{2(x - y)}\right) = \frac{x^2 + xy + y^2 - 3xy}{(x - y) \cdot (x^2 + xy + y^2)} : \\ & \cdot \frac{2x^2 + 2y^2 - x^2 - 2xy - y^2}{2(x - y) \cdot (x + y)} = \frac{x^2 - 2xy + y^2}{(x - y) \cdot (x^2 + xy + y^2)} \cdot \frac{2(x - y) \cdot (x + y)}{x^2 - y^2 - 2xy} = \frac{(x - y)^2}{(x - y) \cdot (x^2 + xy + y^2)} \\ & \cdot \frac{2x^2 - 2y^2}{x^2 - y^2 - 2xy} = \frac{(x - y)^2 \cdot 2(x - y) \cdot (x + y)}{(x - y) \cdot (x^2 + xy + y^2) \cdot (x - y)^2} = \frac{2(x + y)}{x^2 + xy + y^2}. \end{aligned}$$

Answer:  $\frac{2(x + y)}{x^2 + xy + y^2}$  provided that  $x - y \neq 0$ .

Simplify the expression: **2.9**  $\left(\frac{25}{a^2 + 5a + 25} - \frac{2a}{5 - a} - \frac{a^3 + 25a^2}{a^3 - 125}\right) \cdot \left(a - 5 + \frac{15a}{a - 5}\right)$ .

Solution:

$$\begin{aligned} & \left(\frac{25}{a^2 + 5a + 25} - \frac{2a}{5 - a} - \frac{a^3 + 25a^2}{a^3 - 125}\right) \cdot \left(a - 5 + \frac{15a}{a - 5}\right) = \\ & = \left(\frac{25}{a^2 + 5a + 25} + \frac{2a}{a - 5} - \frac{a^2 \cdot (a + 25)}{(a - 5)(a^2 + 5a + 25)}\right) \cdot \frac{a^2 - 10a + 25 + 15a}{a - 5} = \\ & = \frac{25(a - 5) + 2a \cdot (a^2 + 5a + 25) - a^2(a + 25)}{(a - 5) \cdot (a^2 + 5a + 25)} \cdot \frac{a^2 + 5a + 25}{(a - 5)} = \\ & = \frac{25a - 125 + 2a^3 + 10a^2 + 50a - a^3 - 25a^2}{(a - 5) \cdot (a^2 + 5a + 25)} \cdot \frac{a^2 + 5a + 25}{a - 5} = \\ & = \frac{(a^3 - 15a^2 + 75a - 125) \cdot (a^2 + 5a + 25)}{(a - 5) \cdot (a^2 + 5a + 25) \cdot (a - 5)} = \frac{(a - 5)^3}{(a - 5)^2} = a - 5 \end{aligned}$$

When  $a - 5 \neq 0$ .

**2.10** At what natural values R fraction  $\frac{5R^2 + 8R + 12}{R}$  takes natural values?

Solution:

We transform this expression by applying the theorem on the divisibility of the

sum:  $\frac{5R^2 + 8R + 12}{R} = \frac{5R^2}{R} + \frac{8R}{R} + \frac{12}{R}$ .

For any natural values R expression  $5R+8$  is a natural number.

Expression  $\frac{12}{R}$  acquires natural values only with those natural values R, under what 12 is entirely divided into R, that is, with  $R \in \{1;2;3;4;6;12\}$ .

Answer: 1;2;3;4;6;12.

**2.11** Given:  $a \geq \frac{1}{2}, x = 32a - 16$ . Define  $Y = \frac{1}{\sqrt{2a+3-\sqrt{x}}} - \frac{1}{\sqrt{2a+3+\sqrt{x}}}$ .

Solution:

$$\begin{aligned} Y &= \frac{1}{\sqrt{2a+3-\sqrt{x}}} - \frac{1}{\sqrt{2a+3+\sqrt{x}}} = \frac{1}{\sqrt{2a+3-\sqrt{32a-16}}} - \frac{1}{\sqrt{2a+3+\sqrt{32a-16}}} = \\ &= \frac{1}{\sqrt{2a+3-4\sqrt{2a-1}}} - \frac{1}{\sqrt{2a+3+4\sqrt{2a-1}}} = \frac{1}{\sqrt{2a-1+3-4\sqrt{2a-1}}} - \frac{1}{\sqrt{2a-1+3+4\sqrt{2a-1}}} = \\ &= \frac{1}{\sqrt{(\sqrt{2a-1}-2)^2}} - \frac{1}{\sqrt{(\sqrt{2a-1}+2)^2}} = \frac{1}{|\sqrt{2a-1}-2|} - \frac{1}{|\sqrt{2a-1}+2|} = \frac{1}{\sqrt{2a-1}-2} - \frac{1}{\sqrt{2a-1}+2}; \end{aligned}$$

Means,  $a \neq \frac{5}{2}$ .

If a)  $\frac{1}{2} \leq a \leq \frac{5}{2}$ ;  $2 \mid 1 \leq 2a \leq 5 \mid -1$ ;  $0 \leq 2a-1 \leq 4$  | Корінь

extracting the root, we obtain the inequality:

$$0 \leq \sqrt{2a-1} \leq 2 \mid -2 \quad -2 \leq \sqrt{2a-1}-2 \leq 0, \text{ and therefore } |\sqrt{2a-1}-2| = 2-\sqrt{2a-1}.$$

$$Y = \frac{1}{2-\sqrt{2a-1}} - \frac{1}{2+\sqrt{2a-1}} = \frac{2+\sqrt{2a-1}-2+\sqrt{2a-1}}{4-2a+1} = \frac{2\sqrt{2a-1}}{5-2a}.$$

б)  $a > \frac{5}{2}$ ;  $2a > 5 \mid -1$ ;  $2a-1 > 4$ ;  $\sqrt{2a-1} > 2 \mid -2$ ;  $\sqrt{2a-1}-2 > 0$ ;  $|\sqrt{2a-1}-2| = \sqrt{2a-1}-2$ .

$$Y = \frac{1}{\sqrt{2a-1}-2} - \frac{1}{\sqrt{2a-1}+2} = \frac{\sqrt{2a-1}+2-\sqrt{2a-1}+2}{(\sqrt{2a-1})^2-2^2} = \frac{4}{2a-1-4} = \frac{4}{2a-5}.$$

$$\text{Answer: } \begin{cases} \emptyset, \text{ якщо } a = \frac{5}{2}; \\ \frac{2\sqrt{2a-1}}{5-2a}, \text{ якщо } \frac{1}{2} \leq a \leq \frac{5}{2}; \\ \frac{4}{2a-5}, \text{ якщо } a > \frac{5}{2}. \end{cases}$$

## Self-study assignments:

**2.12** At what value of the parameter  $a$  is the square trinomial  $36x^2 - ax + 9$  can be written as the full square of the difference of two monomials?

Answer: 36.

**2.13** Calculate:  $\frac{\sqrt{(b+2)^2-8b}}{\sqrt{b}-\frac{2}{\sqrt{b}}}$ , если  $b = 0,0025$ . Answer:  $-0,05$ .

2.14 Calculate:  $\left(\frac{3}{\sqrt{1+a}} + \sqrt{1-a}\right) : \left(\frac{3}{\sqrt{1-a^2}} + 1\right)$ , если  $a = 0,36$ . Answer: 0,8.

2.15 Calculate:  $1 + \frac{1 + \sqrt{x}}{1 + x + \sqrt{x}} : \frac{1}{x \cdot \sqrt{x} - 1}$ , если  $x = 3,1$ . Answer: 3,1.

2.16 Calculate:  $\left(\frac{x^{\frac{3}{2}} + 8}{x^{\frac{1}{2}} + 2x^{\frac{1}{2}}} - 2x\right) : \frac{x-4}{12} + \frac{48}{x^{\frac{1}{2}} + 2}$ , если  $x = 2,1634$ . Answer: 12.

Simplify expressions:

2.17.  $\frac{x+2}{16} \cdot \left(\frac{4x}{x+2} - \frac{x^3-8}{x^3+8} : \frac{x^2-4}{4x^2-16x+16}\right)$ . Answer:  $\frac{x^3+8x-8}{2 \cdot (x+8)}$ .

2.18.  $\frac{x^2-xy}{y} \cdot \frac{y^2}{x}$ . Answer:  $y \cdot (x-y)$ .

2.19.  $\frac{3a}{e^2} \cdot \frac{ae + e^2}{9}$ . Answer:  $\frac{a \cdot (a+e)}{3e}$ .

2.20.  $\frac{ax-ay}{5x^2y^2} \cdot \frac{5xy}{ey-ex}$ . Answer:  $-\frac{a}{xye}$ .

2.21.  $\frac{xy}{a^2+a^3} \cdot \frac{5xy^2}{x^2y^2}$ . Answer:  $\frac{1}{axy}$ .

2.22.  $\frac{6a}{x^2-x} \cdot \frac{2x-2}{3ax}$ . Answer:  $\frac{4}{x^2}$ .

2.23.  $\frac{9a^2-16e^2}{7a} \cdot \left(\frac{3e-4a}{4e^2-3ae} - \frac{3e+4a}{4e^2+3ae}\right)$ . Answer: 2.

2.24.  $\frac{4xy}{y^2-x^2} : \left(\frac{1}{y^2-x^2} + \frac{1}{x^2+2xy+y^2}\right)$ . Answer:  $2x \cdot (x+y)$ .

2.25.  $\frac{a-2}{a^2+2a} : \left(\frac{a}{a^2-2a} - \frac{a^2+4}{a^3-4a} - \frac{1}{a^2+2a}\right)$ . Answer:  $a-2$ .

2.26. At what natural values of K does the fraction  $\frac{(K-3)^2}{K}$  take natural values?

Answer:  $K \in \{1;9\}$ .