# Section 9
# Trigonometric Equations

The simplest *trigonometric* equations are equations of the type:
$Sinx = a, \quad Cosx = a, \quad tgx = a, \quad ctgx = a.$

The equation $Sinx = a$ has solutions for $|a| \le 1$.

The formula of its roots $x = (-1)^n \arcsin a + \pi n, \quad n \in Z.$

Individual cases: 1). $Sinx = 1,\ x = \frac{\pi}{2} + 2k\pi,\ k \in Z.$

2). $Sinx = -1,\ x = -\frac{\pi}{2} + 2k\pi,\ k \in Z.$

3). $Sinx = 0,\ x = k\pi,\ k \in Z.$

The equation $Cosx = a$ has solutions for $|a| \le 1$, which are determined by the formula $x = \pm \arccos a + 2k\pi$, where $k \in Z.$

Individual cases: 1). $Cosx = 1,\ x = 2k\pi,\ k \in Z.$

2). $Cosx = -1,\ x = \pi + 2k\pi,\ k \in Z.$

3). $Cosx = 0,\ x = \frac{\pi}{2} + k\pi,\ k \in Z.$

The equation $tg = a$ has solutions for all $a \in R, \quad x = arctga + \pi n, \quad \pi \in Z.$

Solution of any trigonometric equation by transformations is reduced to the simplest, which is solved by the general formula, or by the formula of a particular case.

Let's show this with examples.

1). $Sin\frac{2\pi}{x+2} = -1.$

Solution:

$$\frac{2\pi}{x+2} = -\frac{\pi}{2} + 2k\pi,\ k \in Z.\ x + 2 = \frac{2\pi}{-\frac{\pi}{2} + 2k\pi};$$

$$x = \frac{2\pi \cdot 2}{-\pi + 4k\pi} = \frac{4\pi}{4k\pi - \pi} = \frac{4\pi}{\pi(4k-1)} = \frac{4}{4k-1}.$$

Answer: $\frac{4}{4k-1}.$

Given: $(1 + tgx) \cdot (1 + tgy) = 2.$ To find $x + y.$

Solution:

Range of valid values: $Cosx \ne 0, \quad Cosy \ne 0.$

$$1 + tgy + tgx + tgx \cdot tgy = 2;$$

$$tgx + tgy = 2 - 1 - tgx \cdot tgy;$$

$$tgx + tgy = 1 - tgx \cdot tgy | : (1 - tgx \cdot tgy);$$

$$\frac{tgx + tgy}{1 - tgx \cdot tgy} = 1; \quad tg(x + y) = 1; \quad x + y = \frac{\pi}{4} + n\pi, \quad n \in Z.$$

Answer: $x + y = \frac{\pi}{4} + n\pi, \quad n \in Z.$

2). $Sin\left(\frac{2\pi}{3}-\frac{3x}{4}\right)=\frac{1}{2}.$

Solution:

Given the odd function $y=Sinx,$ we have $-Sin\left(\frac{3x}{4}-\frac{2\pi}{3}\right)=-\frac{1}{2};$

$\frac{3x}{4}-\frac{2\pi}{3}=(-1)^n\arcsin\left(-\frac{1}{2}\right)+n\pi,\ \ n\in Z.$

$\frac{3x}{4}-\frac{2\pi}{3}=(-1)^n\cdot\left(\frac{\pi}{6}\right)+n\pi;\ \ \frac{3x}{4}=(-1)^{n+1}\cdot\frac{\pi}{6}+\frac{2\pi}{3}+\pi n\bigg|\cdot\frac{4}{3};\ \ x=(-1)^{n+1}\cdot\frac{9\pi}{9}+\frac{8\pi}{9}+\frac{4\pi n}{3}.$

Answer: $(-1)^{n+1}\cdot\frac{9\pi}{9}+\frac{8\pi}{9}+\frac{4\pi n}{3},\ n\in Z.$

3). $Cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{1}{2}.$

Solution:

Reduce the degree of the left side of the equation:

$\frac{1+Cos2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2}=\frac{1}{2}\bigg|\cdot2;\ \ 1+Cos\left(\frac{\pi}{2}-x\right)=1,\ \ Cos\left(\frac{\pi}{2}-x\right)=1-1;\ \ Cos\left(\frac{\pi}{2}-x\right)=0.$

$\frac{\pi}{2}-x=\frac{\pi}{2}+k\pi,\ \ x=k\pi,\ \ k\in Z.$

Answer: $k\pi.$

4). $tg(2\arccos\cdot3x)=-1.$

Solution:

$2\arccos\cdot3=-\frac{\pi}{4}+k\pi;\ \ \arccos\cdot3x=-\frac{\pi}{8}+\frac{k\pi}{2},\ \ k\in Z.$

$0\le\arccos\cdot2x\le\pi,\ \ 0\le\frac{\pi}{8}+\frac{k\pi}{2}\le\pi\bigg|+\frac{\pi}{8};\ \ \frac{\pi}{8}\le\frac{k\pi}{2}\le\frac{9\pi}{8}\bigg|\cdot\frac{2}{\pi};\ \ \frac{1}{4}\le k\le\frac{9}{4},\ \ K=\{1;\ 2\}.$

If $k=1,$ то $\arccos\cdot3x=-\frac{\pi}{8}+\frac{\pi}{2},\ \ \arccos\cdot3x=\frac{3\pi}{8},\ \ 3x=Cos\frac{3\pi}{8},\ \ x_1=\frac{1}{3}Cos\frac{3\pi}{8};$

If $k=2,$ то $\arccos\cdot3x=-\frac{\pi}{8}+\pi,\ \ \arccos\cdot3x=\frac{7}{8}\pi,\ \ x_2=\frac{1}{3}Cos\frac{7\pi}{8}.$

Answer: $\frac{1}{3}Cos\cdot3\pi/6;\ \frac{1}{3}Cos\cdot7\pi/8.$

5). $Cos\left(2\pi\cdot Cos\frac{3\pi x}{2}\right)=-1.$

Solution:

$2\pi\cdot Cos\frac{3\pi x}{2}=\pi+2k\pi\big|:2\pi.$

$Cos\frac{3\pi x}{2}=\frac{1}{2}+k,\ \ k\in t.$ This equation has solutions provided $\left(\frac{1}{2}+k\right)\le1.$

The last inequality is equivalent to such a double irregularity:

$-1\le\frac{1}{2}+k\le1\bigg|-\frac{1}{2}.\ \ -\frac{3}{2}\le k\le\frac{1}{2};\ \ k=\{-1;\ 0\}.$

At $k=-1$ we have: $Cos\frac{3x}{2}=\frac{1}{2}-1;\quad Cos\frac{3x}{2}=-\frac{1}{2};\quad \frac{3\pi x}{2}=\frac{2\pi}{3};\quad x=\frac{2\pi}{3};\quad \frac{3\pi}{2}=\frac{2\pi}{3}\cdot\frac{2}{3\pi}=\frac{4}{9}$?

At $k=0,\quad Cos\frac{3\pi x}{2}=\frac{1}{2}+0=\frac{1}{2};\quad \frac{3\pi x}{2}=\frac{\pi}{3};$ і $x=\frac{\pi}{3}:\frac{3\pi}{2}=\frac{\pi}{3}\cdot\frac{2}{3\pi}=\frac{2}{9}$.

Answer: $\frac{4}{9};\ \frac{2}{9}$.

We have shown examples of solving practically the simplest trigonometric equations. Although, in principle, every bit more complicated trigonometric equation can be solved in several ways, after all, to facilitate the selection of their way to solve it is advisable to typify.

## Уравнение сводящиеся к квадратным

$\sin^2 x + six + 6 = 0.$

Solution:

We denote $\sin x=t;\ |t|\le 1;\ t^2+3t+6=0.$

$D=9-4\cdot 6=9-24=-15<0;\ t\in\varnothing,\quad x\in\varnothing.$

Answer: $\varnothing$.

$2\sin^2 x+\sin x-1=0.$

Solution:

$\sin x=t,\ |t|\le 1,\ 2t^2+t-1=0.\quad D=1+8=9=3^2.\quad t_1=\frac{-1-3}{4}=-1;\quad t_2=\frac{-1+3}{4}=\frac{1}{2};$

$\sin x=-1;\quad x=-\frac{\pi}{2}+2\pi n,\quad n\in Z.\ \sin x=\frac{1}{2};\quad x=(-1)^n\arcsin\frac{1}{2}+\pi n=(-1)^n\cdot\frac{\pi}{6}+\pi n,\quad n\in Z.$

Answer: $-\frac{\pi}{2}+2\pi n,\ (-1)^n\cdot\frac{\pi}{6}+\pi n,\quad n\in Z.$

$3\sin^2 x-5\sin x-2=0.$

Solution:

$\sin x=t,\ |t|\le 1,\ 3t^2-5t-2=0.\ D=25+24=49=7^2.\quad t_1=\frac{5-7}{6}=-\frac{1}{3};\ t_2=\frac{5+4}{6}=2;$ - не удовлетворяет условие $|t|\le 1.\ \sin x=-\frac{1}{3},$

$x=(-1)^n\arcsin\left(-\frac{1}{3}\right)+\pi n=(-1)^{n+1}\arcsin\frac{1}{3}+\pi n,\quad n\in Z.$

Answer: $(-1)^{n+1}\arcsin\frac{1}{3}+\pi n,\ n\in Z.$

$\cos^2 x-\sin x-1=0.$

Solution:

$\cos^2 x=1-\sin^2 x;\quad 1-\sin^2 x-\sin x-1=0,\quad -\sin^2 x-\sin x=0,$

$-\sin x(\sin x+1)=0\Rightarrow\left[\begin{array}{l}\sin x=0,\\ \sin x+1=0,\end{array}\right.\ \left[\begin{array}{l}x=\pi n,\ n\in Z,\\ \sin x=-1,\end{array}\right.\ \left[\begin{array}{l}x=\pi n,\ n\in Z,\\ x=-\frac{\pi}{2}+2\pi n,\ n\in Z.\end{array}\right.$

Answer: $\pi n,\ -\frac{\pi}{2}+2\pi n,\quad n\in Z.$

$tg^3x + 2tg^2x + 3tgx = 0.$

Solution:

$tgx \cdot (tg^2x + 2tgx + 3) = 0.$ Пускай $tgx = y.$ $y \cdot (y^2 + 2y + 3) = 0.$

$\left[\begin{array}{l} y = 0, \\ y^2 + 2y + 3 = 0. \end{array}\right.$ $\left[\begin{array}{l} y = 0, \\ D < 0, \ y \in \varnothing K. \end{array}\right.$ $tgx = 0, \ x = \pi n, \ n \in Z.$

Answer: $\pi n, \ n \in Z.$

$ctg^2x + 3tgx + 5 = 0.$

Solution:

Let the $ctgx = t.$ $t^2 + 3t + 5 = 0, \ D = 9 - 20 < 0, \ t \in \varnothing$ в $K.$

Answer: $\varnothing.$

$4\sin^2x - \sqrt{2} \cdot \cos^3x = 0.$

Solution:

$\sin^2x = 1 - \cos^2x, \quad 4(1 - \cos^2x) - \sqrt{2}\cos^3x = 0, \quad -\sqrt{2}\cos^3x - 4\cos^2x + 4 = 0 \,|: (-\sqrt{2}),$

$\cos^3x - 2\sqrt{2}\cos^2x - 2\sqrt{2} = 0.$ An interesting solution to this equation:

$\cos^3x + \sqrt{2}\cos^2x + \sqrt{2}\cos^2x + 2\cos x - 2\cos x - 2\sqrt{2} = 0.$

Let's group by two members and take out the common factor from each group outside the brackets:

$(\cos^3 x + \sqrt{2}\cos^2 x) + (\sqrt{2}\cos^2 x + 2\cos x) - (2\cos x + 2\sqrt{2}) = 0,$

$\cos^2(\cos x + \sqrt{2}) + 2\cos x(\cos x + \sqrt{2}) - 2(\cos x + \sqrt{2}) = 0,$

$(\cos x + \sqrt{2})(\cos^2 x + \sqrt{2}\cos x - 2) = 0.$

$\left[\begin{array}{l} \cos x + \sqrt{2} = 0, \\ \cos^2 x + \sqrt{2}\cos x - 2 = 0 \end{array}\right.$ $\left[\begin{array}{ll} \cos x = -\sqrt{2}, & x \in \varnothing \text{ бо не виконується уумова } |\cos x| \le 1. \\ D = 2 + 8 = 10, & \cos x = \dfrac{-\sqrt{2} - \sqrt{10}}{2}, \ x \in \varnothing. \end{array}\right.$

$\cos x = \dfrac{-\sqrt{2} + \sqrt{10}}{2}; \quad x = \pm\arccos\dfrac{-\sqrt{2} + \sqrt{10}}{2} + 2\pi n, \quad n \in Z.$

Answer: $\pm\arccos\dfrac{-\sqrt{2} + \sqrt{10}}{2} + 2\pi n, \ n \in Z.$

$\sin 2x - 2\cos 2x - 3tgx + 2 = 0.$

Solution:

We use a universal substitution, which will reduce the equation to the square.

$\sin 2x = \dfrac{2tgx}{1 + tg^2x}; \quad \cos 2x\dfrac{1 - tg^2x}{1 + tg^2x}; \quad \dfrac{2tgx}{1 + tg^2x} - 2 \cdot \dfrac{1 - tg^2x}{1 + tg^2x} - 3tgx + 2 = 0.$

Let the $tgx = y,$ then $\dfrac{2y}{1 + y^2} - 2 \cdot \dfrac{1 - y^2}{1 + y^2} - 3y + 2 = 0; \quad \dfrac{2y - 2 + 2y^2 - 3y - 3y^3 + 2 + 2y^2}{1 + y^2} = 0;$

$-3y^3 + 4y^2 - y = 0 \,| \cdot (-1); \quad 3y^3 - 4y^2 + y = 0; \quad y \cdot (3y^2 - 4y + 1) = 0;$

$\left[\begin{array}{l} y=0, \\ 3y^2-4y+1=0. \end{array}\right.$ $\left[\begin{array}{l} y=0, \\ D=16-12=4,\ y_2=\dfrac{4+2}{6}=1;\ y_3=\dfrac{4-2}{6}=\dfrac{1}{3}. \end{array}\right.$ $\left[\begin{array}{l} tgx=0, \\ tgx=1, \\ tgx=\dfrac{1}{3}. \end{array}\right.$ $\left[\begin{array}{l} x=\pi n,\ n\in Z, \\ x=\dfrac{\pi}{4}+\pi n,\ n\in Z, \\ x=arctg\left(\dfrac{1}{3}\right)+\pi n,\ n\in Z. \end{array}\right.$

Answer: $\pi n,\ \dfrac{\pi}{4}+\pi n,\ arctg\left(\dfrac{1}{3}\right)+\pi n.$

## Уравнения, однородные относительно $Sinx$ и $Cosx$

$2\sin 5x+5\sin 5x-7\cos 5x=0.$

Solution:

$7\sin 5x-7\cos 5x=0\,|:7\cos 5x;\quad tg5x-1=0;\quad tg5x=1;\quad 5x=arctg1+\pi n;\quad 5x=\dfrac{\pi}{4}+\pi n;$

$x=\dfrac{\pi}{20}+\dfrac{\pi n}{5};\ n\in Z.$

Answer: $\dfrac{\pi}{20}+\dfrac{\pi n}{5};\quad n\in Z.$

Equations of the form $a_0\cdot\sin^n x+a_1\cdot\sin^{n-1}x+...+a_{n-1}\cdot\cos^{n-1}x+a_n\cos^n x=0.$

Де $n\in N,\ a_0,\ a_1,...a_n$ – numbers are called homogeneous with respect to $Sinx;\ Cosx.$

$15\sin^2 x+15\sin x\cdot\cos x-10\cos^2 x=10.$

Solution:

$10=10\cdot 1=10\cdot\left(\sin^2 x+\cos^2 x\right)=10\sin^2 x+10\cos^2 x.$

$15\sin^2 x+15\sin x\cdot\cos x-10\cos^2 x-10\sin^2 x-10\cos^2 x=0,$

$5\sin^2 x+15\sin x\cdot\cos x-20\cos^2 x=0\,|:5\cos^2 x.$

$tg^2x+3tgx-4=0$ $\left[\begin{array}{l} tgx=-4, \\ tgx=1. \end{array}\right.$ $\left[\begin{array}{l} x=arctg(-4)+\pi n,\ n\in Z, \\ x=arctg1+\pi n,\ n\in Z. \end{array}\right.$

$x=-arctg4+\pi n,\ n\in Z.\quad x=\dfrac{\pi}{n}+\pi n,\ n\in Z.$

Answer: $arctg4+\pi n,\ \dfrac{\pi}{4}+\pi n,\ n\in Z.$

$3\sin x+4\cos^2\dfrac{x}{2}=2\sin x+10\cos^2\dfrac{x}{2}-2.$

Solution:

$3\sin x+4\cos^2\dfrac{x}{2}-2\sin x-10\cos^2\dfrac{x}{2}+2=0.$

$\sin x+6\cos^2\dfrac{x}{2}+2=0;\quad \sin x=\sin 2\left(\dfrac{x}{2}\right)=2\sin\dfrac{x}{2}\cdot\cos\dfrac{x}{2}.$

$2=2\cdot 1=2\left(\cos^2\dfrac{x}{2}+\sin^2\dfrac{x}{2}\right)=2\cos^2\dfrac{x}{2}+2\sin^2\dfrac{x}{2};$

$2\sin\dfrac{x}{2}\cdot\cos\dfrac{x}{2}-6\cdot\cos^2\dfrac{x}{2}+2\cos^2\dfrac{x}{2}+2\sin^2\dfrac{x}{2}=0;$

$$2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cdot\cos\frac{x}{2}-4\cos^2\frac{x}{2}=0 \Big|: 2\cos^2\frac{4}{2};$$

$$tg^2\frac{x}{2}+tgx-2=0, \quad \left[\begin{array}{l} tg\frac{x}{2}=-2, \\ tg\frac{x}{2}=1. \end{array}\right. \quad \left[\begin{array}{l} -\frac{x}{2}=arctg(-2)+\pi n,\ n\in Z. \\ \frac{x}{2}=arctg1+\pi n,\ n\in Z. \end{array}\right. \quad \left[\begin{array}{l} x=-2arctg2+2\pi n,\ n\in Z. \\ x=\frac{\pi}{2}+\pi n,\ n\in Z. \end{array}\right.$$

Answer: $-2arctg2+2\pi n,\ \frac{\pi}{2}+\pi n,\ n\in Z.$

$$\frac{1}{2}\sin 2x+3\cos^2 x=0.$$

Solution:

$$\frac{1}{2}\cdot 2\sin x\cdot\cos x+3\cos^2 x=0, \quad \left[\begin{array}{l} \cos x=0, \\ \sin x+3\cos x=0. \end{array}\right. \quad x=\frac{\pi}{2}+\pi n,\quad n\in Z;\quad tgx+3=0,$$

$$\cos x\cdot(\sin x+3\cos x)=0$$

$$tgx=-3,\quad x=-arctg3+\pi n.$$

Answer: $\frac{\pi}{2}+\pi n,\ -arctg3+\pi n,\quad n\in Z.$

***Task.*** One of the corners of a right-angled triangle satisfies the condition
$\sin^3 x+\sin x\cdot\sin 2x-3\cos^3 x=0.$
Prove that a triangle is isosceles.

Solution:

$$\sin^3 x+\sin x\cdot\sin 2x-3\cos^3 x=0 \Big|: \cos^3 x;\qquad tg^3x+\frac{2\sin^2 x\cdot\cos x}{\cos^3 x}-3=0;$$

$$tg^3x+2tg^2x-3=0;\quad tg^3x-tg^2x+3tg^2x-3=0;\quad (tg^3x-tg^2x)+(3tg^2x-3)=0;$$

$$tg^2x(tgx-1)+3(tg^2x-1)(tgx+1)=0;\quad (tgx-1)(tg^2x+3tgx+3)=0;$$

$$\left[\begin{array}{l} tgx-1=0, \\ tg^2x+3tgx+3=0 \end{array}\right. \quad \left[\begin{array}{l} tgx=1, \\ D=9-12<0 \end{array}\right. \quad \left[\begin{array}{l} x=\frac{\pi}{4}+\pi n, \\ x\in\varnothing. \end{array}\right.$$

Since one of the acute angles of a right triangle is equal to $\frac{\pi}{4}$.

So the triangle is isosceles.

$$\sin^4 x\cdot\cos^2 x-2\sin^3 x\cdot\cos^3 x-\sin^2 x\cdot\cos^4 x+2\sin x\cdot\cos^5 x=0.$$

Solution:

These are homogeneous equations of the 6th degree. Take the common factor out of the parentheses:

$$\sin x\cdot\cos^2 x\cdot(\sin^{\backslash 3} x-2\sin^2 x\cdot\cos x-\sin x\cdot\cos^2 x+2\cos^3 x)=0.$$

Let's solve the set of equations:

$$\left[\begin{array}{l} \sin x=0,\ x=\pi n,\ n\in Z. \\ \cos^2 x=0,\ \cos x=0,\ x=\frac{\pi}{2}+k\pi,\ k\in Z. \\ \sin^{\backslash 3} x-2\sin^2 x\cdot\cos x-\sin x\cdot\cos^2 x+2\cos^3 x=0. \end{array}\right.$$

The third equation together - a homogeneous equation of the third degree. Divide its left and right parts into $\cos^3 x$:

$$tg^3x+2tg^2x-tgx+2=0;\quad tg^2x\cdot(tgx+2)-(tgx-2)=0;\quad (tgx-2)\cdot(tg^2x-1)=0;$$

$(tgx-2)\cdot(tgx-1)\cdot(tgx+1)=0.$

$$\left[\begin{array}{l} tgx-2=0, \\ tgx-1=0, \\ tgx+1=0. \end{array}\right. \quad \left[\begin{array}{l} tgx=2, \quad x=arctg2+\pi n, \ n\in Z. \\ tgx=1, \quad x=\dfrac{\pi}{4}+\pi n, \ n\in Z. \\ tgx=-1. \quad x=-\dfrac{\pi}{4}+\pi n, \ n\in Z. \end{array}\right.$$

Answer: $\pi n, \ \frac{\pi}{2}+\pi n, \ \text{arctg}2+\pi n, \ \frac{\pi}{4}+\pi n, \ -\frac{\pi}{4}+\pi n, \ n\in Z.$

$\sin^2 x\cdot\cos x-3\cos^3 x=\sin x-2\cos x.$

Solution:

This equation can easily be reduced to a homogeneous third degree equation. Let's transform its right side:

$\sin x-2\cos x=(\sin x-2\cos x)\cdot 1=(\sin x-2\cos x)\cdot(\sin^2 x+\cos^2 x)=\sin^3 x+\sin x\cdot\cos^2 x-$
$-2\cos x\cdot\sin^2 x-2\cos^3 x.$

Then this equation takes the form:

$\sin^2 x\cdot\cos x-3\cos^3 x=\sin^3 x+\sin x\cos^2 x-2\cos x\cdot\sin^2 x-2\cos^3 x,$

$\sin^2 x\cdot\cos x-3\cos^3 x-\sin^3 x-\sin x\cdot\cos^2 x+2\cos x\cdot\sin^2 x+2\cos^3 x=0,$

$3\sin^2 x\cdot\cos x-\sin x\cdot\cos^2 x-\cos^3 x-\sin^3 x=0 \,|: (-\cos^3 x),$

$3tg^2x+tgx+1+tg^3x=0; \quad tg^3x-3tg^2x+tgx+1=0.$

The free term of equation 1 has a divisor $1;-1$. 1 – the root of this equation, because $1-3+1+1=0.$

$$\begin{array}{l|l} tg^3x-3tg^2x+tgx+1 & tgx-1 \\ \cline{2-2} -\ tg^3x-tg^2x & tg^2x-2tgx-1 \\ \hline \quad -\ \begin{array}{l} -2tg^2x+tgx \\ -2tg^2x+tgx \end{array} & \\ \hline \qquad -\ \begin{array}{l} -tgx+1 \\ -tgx+1 \end{array} & \\ \hline \qquad\qquad 0. & \end{array}$$

$$\left[\begin{array}{l} tgx=1, \\ tg^2x-2tgx-1=0. \end{array}\right. \qquad x=\frac{\pi}{4}+\pi n, \ n\in Z.$$

Let's solve the second equation of the population: $D=4+4=8.$

$$\left[\begin{array}{l} tgx=\dfrac{2-\sqrt{8}}{2}=\dfrac{2-2\sqrt{2}}{2}=1-\sqrt{2}, \\ tgx=\dfrac{2+2\sqrt{2}}{2}=1+\sqrt{2}. \end{array}\right. \quad \left[\begin{array}{l} x=arctg(1-\sqrt{2})+\pi n, \ n\in Z. \\ x=arctg(1+\sqrt{2})+\pi n, \ n\in Z. \end{array}\right.$$

Answer: $\frac{\pi}{4}+\pi n, \quad arctg(1-\sqrt{2})+\pi n, \quad arctg(1+\sqrt{2})+\pi n, \quad n\in Z.$

## Уравнения линейные относительно $\sin x; \ \cos x.$

Equations of the form $a \cdot \sin x + b \cdot \cos x = c,$ де $a, b : c -$
Numbers are called linear equations with respect to $\sin x : \cos x.$
If at least one of the coefficients is equal to zero, then the equation is reduced to a single of the already considered types of equations. If $a \neq 0, \; b \neq 0, \; c \neq 0,$ then we show some methods for solving this equation.

$3\sin x + 2\cos x = 1.$

Solution:

$\sin x = \sin\left(2 \cdot \frac{x}{2}\right) = 2\sin\frac{x}{2} \cdot \cos\frac{x}{2},$

$\cos x = \cos\left(2 \cdot \frac{x}{2}\right) = \cos^2\frac{x}{2} - \sin^2\frac{x}{2},$

$1 = \sin^2\frac{x}{2} + \cos^2\frac{x}{2}.$

$3 \cdot 2\sin\frac{x}{2} \cdot \cos\frac{x}{2} + 2\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right) = \cos^2\frac{x}{2} + \sin^2\frac{x}{2},$

$6\sin\frac{x}{2} \cdot \cos\frac{x}{2} + 2\cos^2\frac{x}{2} - 2\sin^2\frac{x}{2} - \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 0,$

$-3\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 6\sin^2\frac{x}{2} \cdot \cos\frac{x}{2} = 0 \Big| : \left(-\cos^2\frac{x}{2}\right),$

$3tg^2\frac{x}{2} - 6tg\frac{x}{2} - 1 = 0, \quad D = 36 + 12 = 48.$

$tg\frac{x}{2} = \frac{6 - \sqrt{48}}{2 \cdot 3} = \frac{6 - 2\sqrt{12}}{6} = \frac{3 - \sqrt{12}}{3}; \qquad tg\frac{x}{2} = \frac{3 + \sqrt{12}}{3}.$

$\frac{x}{2} = arctg\frac{3 - \sqrt{12}}{3} + \pi n^3; \quad \frac{x}{2} = arctg\frac{3 + \sqrt{12}}{3} + \pi n, \; n \in Z.$

$x = 2arctg\frac{3 - \sqrt{12}}{3} + 2\pi n, \quad x = 2arctg\frac{3 + \sqrt{12}}{3} + 2\pi n, \; n \in Z.$

Answer: $2arctg\frac{3 - \sqrt{12}}{3} + 2\pi n, \quad 2arctg\frac{3 + \sqrt{12}}{3} + 2\pi n, \; n \in Z.$

$\sqrt{3}\sin x - \cos x = \sqrt{3}.$

Solution:

Divide both sides of the equation by 2: $\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{\sqrt{3}}{2}.$

Replace $\frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}, \; \frac{1}{2} = \sin\frac{\pi}{6}.$ Then $\cos\frac{\pi}{6} \cdot \sin x - \sin\frac{\pi}{6} \cdot \cos x = \frac{\sqrt{3}}{2}; \quad \sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2};$

$x - \frac{\pi}{6} = (-1)^n \arcsin\frac{\sqrt{3}}{2} + \pi n, \; n \in Z; \quad x - \frac{\pi}{6} = (-1)^n\frac{\pi}{3} + \pi n, \quad x = \frac{\pi}{6} + (-1)^n\frac{\pi}{3} + \pi n, \; n \in Z.$

Answer: $\frac{\pi}{6} + (-1)^n\frac{\pi}{3} + \pi n, \; n \in Z.$

$4\cos x - 4\sin x = \sqrt{8}.$

Solution:

On the left side of the equation put out of brackets $4\sqrt{2}$ :

$$4\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)=\sqrt{8};$$

$$4\sqrt{2}\left(\cos\frac{\pi}{4}\cos x-\sin\frac{\pi}{4}\sin x\right)=\sqrt{8};\quad 4\sqrt{2}\cdot\cos\left(x+\frac{\pi}{4}\right)=\sqrt{8};\quad \cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{8}}{4\sqrt{2}};$$

$$\cos\left(x+\frac{\pi}{4}\right)=\frac{2\sqrt{2}}{4\sqrt{2}};\quad \cos\left(x+\frac{\pi}{4}\right)=\frac{1}{2};\quad x=-\frac{\pi}{4}\pm\frac{\pi}{3}+2\pi n,\ n\in Z.$$

Answer: $-\frac{\pi}{4}\pm\frac{\pi}{3}+2\pi n,\ n\in Z.$

## Equation whose left sides are expressed by products of trigonometric functions, and the right ones are equal to zero

Perhaps the greatest difficulties are caused by the construction of answers of this type of trigonometric equations. Here the following indication helps.:

1). Use the condition of equality of the product to zero;

2). Solve the set of generated trigonometric equations;

3). From the found set of solutions, filter out extraneous roots, that is, those for which some of the factors in the equation express the content;

4). if some of the equations of the set have two identical roots, then in response take one of them.

Let's show this with examples.

$$\sin 2x\cdot tg3x\cdot ctg\left(x-\frac{\pi}{3}\right)=0.$$

Solution:

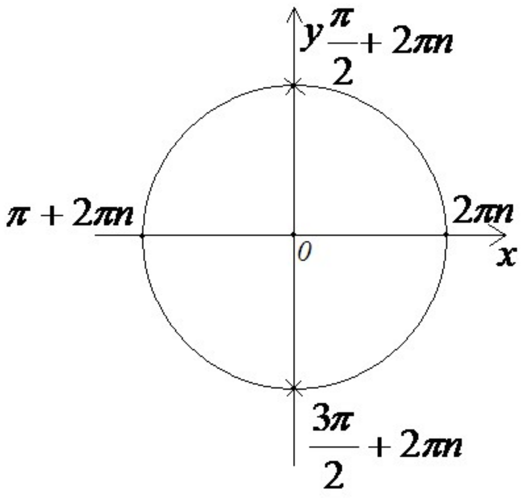
Using the conditions for the equality of the product to zero, we obtain the following set of equations:

$$\left[\begin{array}{l}\sin 2x=0,\\ tg3x=0,\\ ctg\left(x-\frac{\pi}{3}\right)=0,\end{array}\right.\quad \left[\begin{array}{l}2x=k\pi,\\ 3x=k\pi,\\ x-\frac{\pi}{3}=\frac{\pi}{2}+\pi n,\end{array}\right.\quad \left[\begin{array}{l}x=\frac{k\pi}{2},\ n\in Z.\\ x=\frac{k\pi}{3},\ n\in Z.\\ x=\frac{5\pi}{6}+k\pi,\ n\in Z.\end{array}\right.$$

Let us represent the solutions of each equation of the set by the corresponding angles on the unit circle.

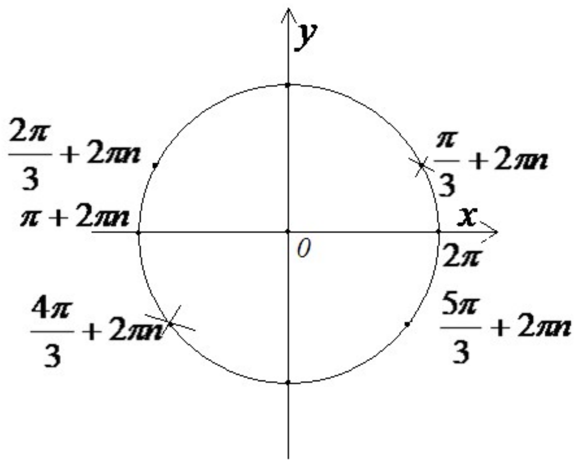
$x=\frac{k\pi}{2}$,

| $K$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $X$ | 0 | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $2{,}5\pi$ | $3\pi$ | $3{,}5\pi$ |



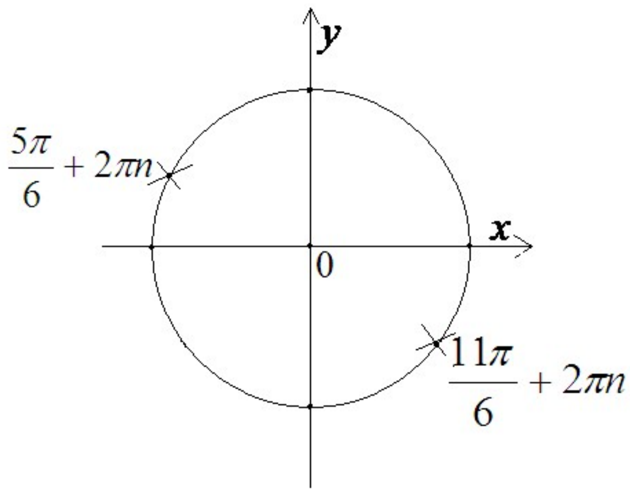

| K | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $x$ | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4}{3}\pi$ | $\frac{5}{3}\pi$ | $2\pi$ | $\frac{7}{3}\pi$ |

$$x=\frac{K\pi}{3}$$




$$x=\frac{5\pi}{6}+K\pi$$

| K | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $x$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{6}$ | $\frac{17\pi}{6}$ | $\frac{23\pi}{6}$ | $\frac{29\pi}{6}$ | $\frac{35}{6}\pi$ | $\frac{41\pi}{6}$ | $\frac{47}{6}\pi$ |




Numbers $x=\frac{\pi}{2}+2\pi n,\ x=\frac{3\pi}{2}+2\pi n,\ x=\frac{5\pi}{6}+2\pi n,\ x=\frac{11\pi}{6}+2\pi n-$ extraneous roots, since with them $tg3x$ does not exist.

Numbers $x=\frac{\pi}{3}+2\pi n,\ x=\frac{4\pi}{3}+2\pi n-$ also extraneous roots, since with them $ctg\left(x-\frac{\pi}{3}\right)$ does not exist. In each figure, we emphasize these dots. In response, we write down the dots that remained in the figures.

Answer: $x=m\pi,\ x=\frac{2\pi}{3}+m\pi,\ m\in Z.$

$\cos 3x-\cos 7x=0.$

Solution:

$2\sin\frac{3x+7x}{2}\cdot\sin\frac{7x-3x}{2}=0,$

$2\sin 5x\cdot\sin 2x=0,$

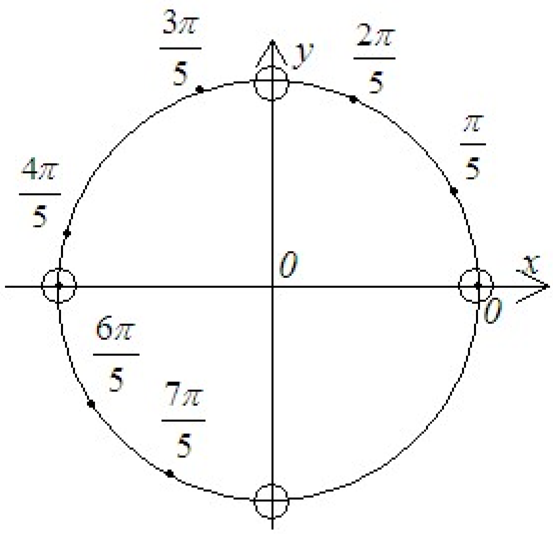
$\left[\begin{array}{l}\sin 5x=0,\\ \sin 2x=0.\end{array}\right.\ \left[\begin{array}{l}5x=k\pi,\\ 2x=k\pi.\end{array}\right.\ \left[\begin{array}{l}x=\frac{k\pi}{5},\\ x=\frac{k\pi}{2}.\end{array}\right.$

$x=\frac{k\pi}{5}$

| K | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| X | 0 | $\frac{\pi}{5}$ | $\frac{2\pi}{5}$ | $\frac{3\pi}{5}$ | $\frac{4\pi}{5}$ | $\pi$ | $\frac{6\pi}{5}$ | $\frac{7\pi}{5}$ |

$x=\frac{k\pi}{2}$

| K | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| X | 0 | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ | $3\pi$ | $3{,}5\pi$ |




The values $x = \pi n$ belong to both sets of solutions. Therefore, we remove these values from one of them, for example, from the set $x = \frac{k\pi}{2}$.

Then the sought solutions will be of the form: ( $x = \frac{\pi}{2} + \pi n.$ after filtration).

$$x = \frac{m\pi}{5}; \quad x = \frac{\pi}{2} + m\pi, \; m \in Z.$$

Answer: $x = \frac{m\pi}{5}; \quad x = \frac{\pi}{2} + m\pi, \; m \in Z.$

$\cos^2 x + \cos^2 2x = \cos^2 3x + \cos^2 4x.$

Solution:

$$\cos^2 x = \frac{1+\cos 2x}{2}, \quad \cos^2 2x = \frac{1+\cos 4x}{2}, \quad \cos^2 3x = \frac{1+\cos 6x}{2}, \quad \cos^2 4x = \frac{1+\cos 8x}{2},$$

$$\frac{1+\cos 2x}{2} + \frac{1+\cos 4x}{2} = \frac{1+\cos 6x}{2} + \frac{1+\cos 8x}{2} \Big| \cdot 2;$$

$$1 + \cos 2x + 1 + \cos 4x = 1 + \cos 6x + 1 + \cos 8x;$$

$$2\cos\frac{2x+4x}{2} \cdot \cos\frac{2x-4x}{2} = 2\cos\frac{6x+8x}{2} \cdot \cos\frac{6x-8x}{2} \Big| : 2;$$

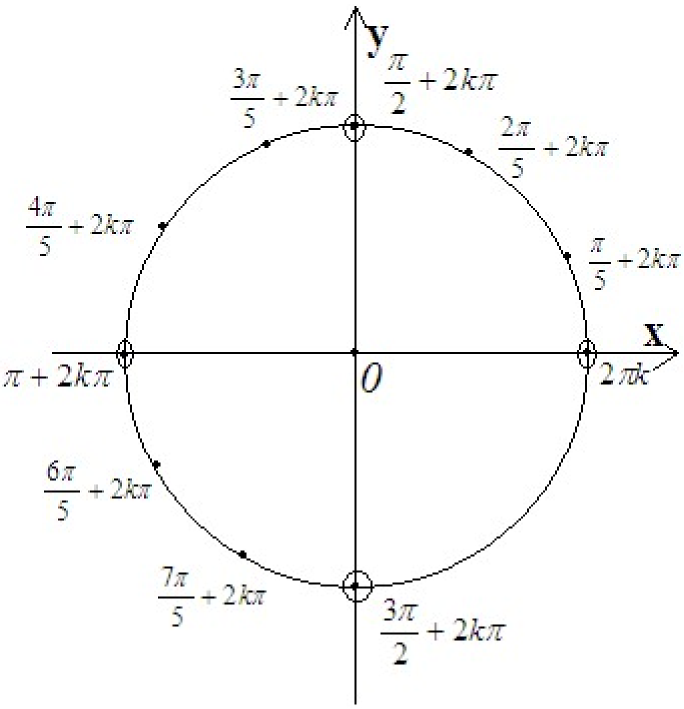
$$\cos 3x \cdot \cos(-x) = \cos 17x \cdot \cos(-x);$$

$$\cos 3x \cdot \cos x - \cos 7x \cdot \cos x = 0;$$

$$\cos x \cdot (\cos 3x - \cos 7x) = 0; \quad \cos \cdot 2\sin\frac{3x+7x}{2}, \quad \sin\frac{7x-3x}{2} = 0$$

$$2\cos \cdot \sin 5x \cdot \sin 2x = 0$$

$$\left[\begin{array}{l} \cos x = 0, \\ \sin 5x = 0, \\ \sin 2x = 0. \end{array}\right. \quad \left\{\begin{array}{l} x = \frac{\pi}{2} + k\pi, \; k \in Z \\ 5x = \pi n, \\ 2n = \pi n \end{array}\right. \quad \left[\begin{array}{l} x = \frac{\pi}{2} + k\pi, \\ x = \frac{k\pi}{5}, \\ x = \frac{k\pi}{2}. \end{array}\right.$$




$x=\frac{k\pi}{5}+k\pi$

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\frac{\pi}{2}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{2}$ | $\frac{7\pi}{2}$ | $\frac{9\pi}{2}$ | $\frac{11\pi}{2}$ | $\frac{13\pi}{2}$ | $\frac{15\pi}{2}$ |

$x=\frac{k\pi}{5}$

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 0 | $\frac{\pi}{5}$ | $\frac{2\pi}{5}$ | $\frac{3\pi}{5}$ | $\frac{4\pi}{5}$ | $\pi$ | $\frac{6\pi}{5}$ | $\frac{7\pi}{5}$ |

$x=\frac{k\pi}{2}$

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 0 | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ | $3\pi$ | $\frac{7\pi}{2}$ |

Of the multitude $x=\frac{k\pi}{2}$ exclude numbers $2\pi n,\quad \frac{\pi}{2}+2\pi n,\quad \pi+2k\pi,\quad \frac{3\pi}{2}+2\pi n.$

Answer: $x=\frac{\pi}{2}+k\pi,\quad x=\frac{k\pi}{5},\quad k\in Z.$

$tg7x-tgx=0.$

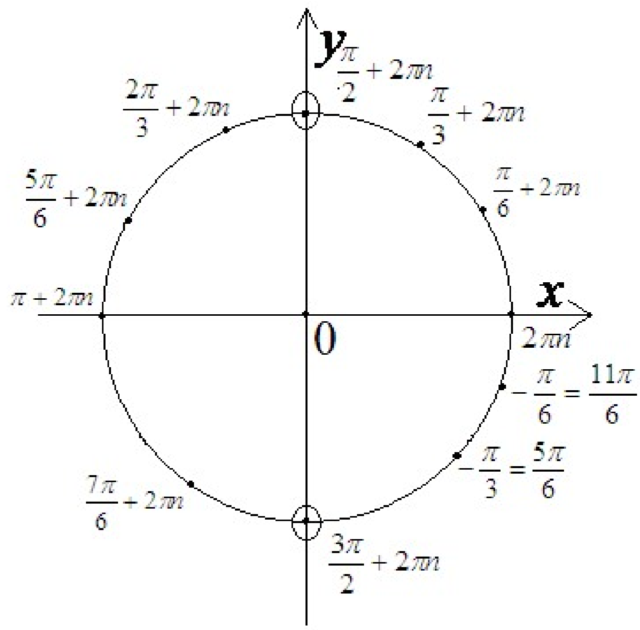
Solution:

$$\frac{\sin 7x}{\cos 7x}-\frac{\sin x}{\cos x}=0,\quad \frac{\sin x\cdot\cos x-\cos 7x\cdot\sin x}{\cos 7x\cdot\cos x}=0,\quad \frac{\sin(7x-x)}{\cos 7x\cdot\cos x}=0,\quad \frac{\sin 6x}{\cos 7x\cdot\cos x}=0.$$

$$\begin{cases}\sin 6x = 0,\\ \cos 7x \neq 0,\\ \cos x \neq 0.\end{cases} \quad \begin{cases}6x = \pi n,\\ 7x \neq \dfrac{\pi}{2} + \pi n,\\ x \neq \dfrac{\pi}{2} + \pi n.\end{cases} \quad \begin{cases}x = \dfrac{\pi n}{6},\\ x \neq \dfrac{\pi}{14} + \dfrac{\pi}{7} n,\\ x \neq \dfrac{\pi}{2} + \pi n.\end{cases}$$

$x = \dfrac{\pi n}{6}.$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $n$ |
| 0 | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ | $x$ |




Extraneous root $x = \dfrac{\pi}{2} + \pi n$

remove from the series $x = \dfrac{\pi n}{6}$.

Answer: $x = \pm\dfrac{\pi}{6} + \dfrac{m\pi}{2}$,

$x = \pi m, \quad n \in Z.$

$tg5x - tg4x - tgx = 0.$

Solution:

From the formula for the tangent of the difference, we have:

$tg(5x - 4x) = \dfrac{tg5x - tg4x}{1 + tg5x \cdot tg4x}.$

Then $tg5x - tg4x = tg(5x - 4x) \cdot (1 + tg5x \cdot tg4x).$

6 44 7^x 4 48

$tg(tg5x - tg4x) \cdot (1 + tg5x \cdot tg4x) - tgx = 0,$

$tg \cdot (1 + tg5x \cdot tg4x) - tgx = 0,$

$tgx + tgx \cdot tg5x \cdot tg4x - tgx = 0,$

$tgx \cdot tg4x \cdot tg5x = 0.$

$x = \pi n.$

$n$ 1 2 3 4 5 6 7 0

$x$ $\pi$ 0 $\pi$ 0 $\pi$ 0 $\pi$ 0

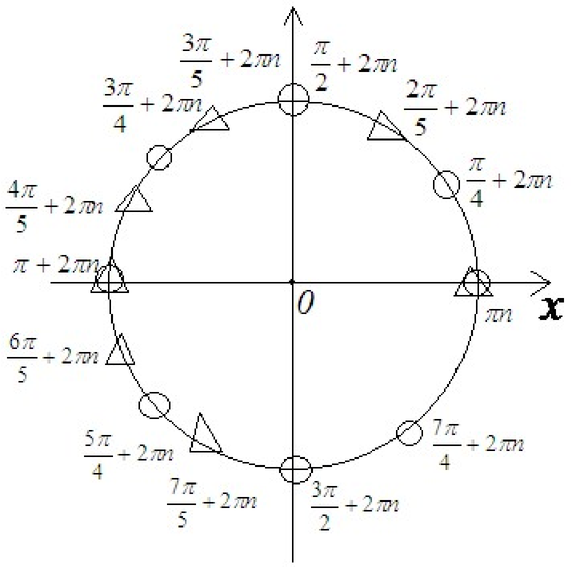
$x = \frac{\pi n}{4}$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *n* | *1* | *2* | *3* | *4* | *5* | *6* | *7* | *0* |
| *x* | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{6\pi}{2}$ | $\frac{7\pi}{4}$ | *0* |

$x = \frac{\pi n}{5}$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $x$ | 0 | $\frac{\pi}{5}$ | $\frac{2\pi}{5}$ | $\frac{3\pi}{5}$ | $\frac{4\pi}{5}$ | $\pi$ | $\frac{6\pi}{5}$ | $\frac{7\pi}{5}$ |

$$\left[\begin{array}{l} tgx = 0, \\ tg4x = 0, \\ tg5x = 0. \end{array}\right. \left[\begin{array}{l} x = \pi n, \\ 4x = \pi n, \\ 5x = \pi n. \end{array}\right. \left[\begin{array}{l} x = \pi n, \\ x = \frac{\pi n}{4}, \\ x = \frac{\pi n}{5}. \end{array}\right.$$




From the set of solutions in the population, it is necessary to "filter" $x = \pi n$, because it is repeated in two other series. From the series $x = \frac{\pi n}{4}$ exclude too $x = \pi n$.

Answer: $x = \frac{m\pi}{5}, \quad n \in Z. \quad x = \frac{\pi}{4} + \frac{m\pi}{2}, \quad m \in Z.$

# Solving more complex trigonometric equations

$$\cos^4 x + \cos^4\left(x - \frac{\pi}{4}\right) = \frac{5}{4}.$$

Solution:

$$\cos^4 x = \left(\cos^2 x\right)^2 = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1 + 2\cos 2x + \cos^2 2x}{4}.$$

$$\cos^4\left(x-\frac{\pi}{4}\right)=\left(\cos^2\left(x-\frac{\pi}{4}\right)\right)^2=\left(\frac{1+\cos\left(2x-\frac{\pi}{2}\right)}{2}\right)^2=\left(\frac{1+\cos\left(\frac{\pi}{2}-2x\right)}{2}\right)^2=$$

$$=\left(\frac{1+\sin 2x}{2}\right)^2=\frac{1+2\sin 2x+\sin^2 2x}{4}=\frac{2+2\sin 2x+\cos 2x+\overbrace{\left(\sin^2 2x+\cos^2 2x\right)}^{1}}{4}=\frac{5}{4}.$$

$$\frac{3+2(\sin 2x+\cos 2x)}{4}=\frac{5}{4}\Big|\cdot 4.\quad 3+2(\sin 2x+\cos 2x)=5.\quad 2(\sin 2x+\cos 2x)=5-3;$$

$$\sin 2x+\cos 2x=1.\quad 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}-\cos^2\frac{x}{2}-\sin^2\frac{x}{2}=0.$$

$$2\sin\frac{x}{2}\cdot\cos\frac{x}{2}-2\sin^2\frac{x}{2}=0.\quad 2\sin\frac{x}{2}\cdot\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)=0.$$

$$\left[\begin{array}{l}\sin\frac{x}{2}=0,\\ \cos\frac{x}{2}-\sin\frac{x}{2}=0.\end{array}\right.\quad \left[\begin{array}{l}\frac{x}{2}=\pi n,\\ \sin\frac{x}{2}=\cos\frac{x}{2}\Big|:\cos\frac{x}{2}.\end{array}\right.\quad \left[\begin{array}{l}x=2\pi n,\\ tg\frac{x}{2}=1.\end{array}\right.\quad \left[\begin{array}{l}x=2\pi n,\\ \frac{x}{2}=\frac{\pi}{4}+\pi n.\end{array}\right.\quad \left[\begin{array}{l}x=2\pi n,\\ x=\frac{\pi}{2}+2\pi n.\end{array}\right.$$

Answer: $2\pi n;\ \frac{\pi}{2}+2\pi n,\ n\in Z.$

$3ctg3x-4ctg4x=tg3x.$

Solution:

A method that is very rarely used when solving trigonometric equations. Add to both sides of the equation $ctg3x$:

$$3ctg3x+4ctg4x+ctg3x=tg9x+ctg3x;\qquad 4ctg3x-4ctg4x=tg3x+ctg3x;$$

$$4\cdot\left(\frac{\cos 3x}{\sin 3x}-\frac{\cos 4x}{din4x}\right)=\frac{\sin 3x}{\cos 3x}+\frac{\cos 3x}{\sin 3x};$$

$$4\cdot\frac{\sin 4x\cdot\cos 3x-\cos 4x\cdot\sin 3x}{\sin 3x\cdot\sin 4x}=\frac{\sin^2 3x+\cos^2 3x}{\cos 3x\cdot\sin 3x};\qquad 4\cdot\frac{\sin(4x-3x)}{\sin 3x\cdot\sin 4x}=\frac{1}{\cos 3x\cdot\sin 3x};$$

$$\frac{4\sin x}{\sin 3x\cdot\sin 4x}=\frac{1}{\cos 3x\cdot\sin 3x};$$

Using the main property of proportion, we have:

$$4\sin x\cdot\cos 3x\cdot\sin 3x=\sin 3x\cdot\sin 4x\big|:\sin 3x;$$

$$4\sin x\cdot\cos 3x=\sin 4x;$$

On the left side of the equation we transform the product of trigonometric functions into the sum:

$$4\cdot\left(\frac{1}{2}\left(\sin(x+3x)+\sin(x-3x)\right)\right)=2\sin 4x+2\sin(-2x)=2\sin 4x-2\sin 2x;$$ Тоді:

$$2\sin 4x-2\sin 2x=\sin 4x;\quad \sin 4x-2\sin 2x=0;\quad 2\sin 2x\cdot\cos 2x-2\sin 2x=0;$$

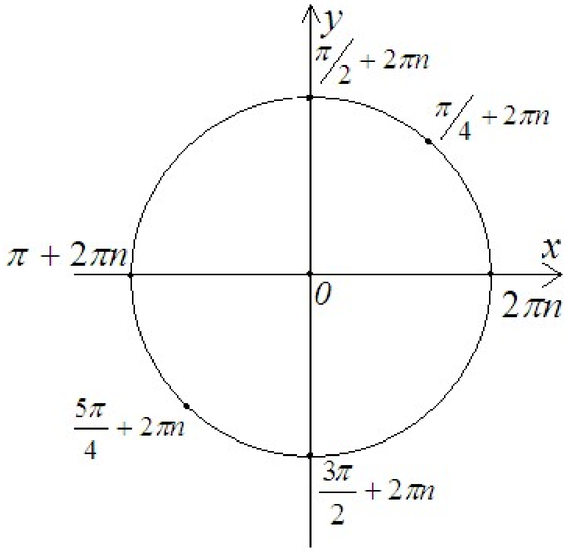
$$2\sin 2x\cdot(\cos 2x-1)=0;$$

$$\left[\begin{array}{l}\sin 2x=0,\\ \cos 2x=1\end{array}\right.\quad \left[\begin{array}{l}2x=\pi n,\\ 2x=\frac{\pi}{2}+2\pi n\end{array}\right.$$

$x = \frac{\pi n}{2}$,

$x = \frac{\pi}{4} + \pi n.$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $x$ | $\pi/4$ | $\frac{5\pi}{4}$ | $\frac{9\pi}{4}$ | $\frac{13\pi}{4}$ | $\frac{17\pi}{4}$ | $\frac{21\pi}{4}$ | $\frac{25\pi}{4}$ | $\frac{29\pi}{4}$ |




If $x = 0$, then $ctg3x$ does not exist.

If $x = \frac{\pi}{4}$, then $ctg4x$ does not exist.

Answer: $x \in \varnothing$.

$$\sqrt{3} - \frac{3ctgx}{1 - 2\sqrt{\cos x}} + 2\sqrt{3\cos x} = 0.$$

Solution:

$$\sqrt{3}\left(1 - \frac{\sqrt{3}ctgx}{1 - 2\sqrt{\cos x}} + 2\sqrt{\cos x}\right) = 0, \quad 1 - \frac{\sqrt{3}ctgx}{1 - 2\sqrt{\cos x}} + 2\sqrt{\cos x} = 0 \Big| : \left(1 - 2\sqrt{\cos x}\right);$$

$$\left(1 + 2\sqrt{\cos x}\right)\cdot\left(1 - 2\sqrt{\cos x}\right) - \sqrt{3}\cdot ctgx = 0, \quad 1 - 4\cos x - \sqrt{3}\cdot ctgx = 0,$$

$$1 - 4\cos x - \sqrt{3}\cdot\frac{\cos x}{\sin x} = 0 \Big| : \sin x; \quad \sin x - 4\cos x\cdot\sin x - \sqrt{3}\cos x = 0;$$

$$\sin x - 2\sin 2x - \sqrt{3}\cos x = 0 \Big| : 2; \quad \frac{1}{2}\cdot\sin x - \sin 2x - \frac{\sqrt{3}}{2}\cos x = 0;$$

$$\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right) - \sin 2x = 0; \quad \cos\frac{\pi}{3}\cdot\sin x - \sin\frac{\pi}{3}\cdot\cos x - \sin 2x = 0;$$

$$2\sin\frac{x - \frac{\pi}{3} - 2x}{2}\cdot\cos\frac{x - \frac{\pi}{3} + 2x}{2} = 0; \quad 2\sin\left(-\frac{x}{2} - \frac{\pi}{6}\right)\cdot\cos\left(\frac{3x}{2} - \frac{\pi}{6}\right) = 0;$$

$$-2\sin\left(\frac{x}{2} + \frac{\pi}{6}\right)\cdot\cos\left(\frac{3x}{2} - \frac{\pi}{6}\right) = 0 \Big| : (-2); \quad \sin\left(\frac{x}{2} + \frac{\pi}{6}\right)\cdot\cos\left(\frac{3x}{2} - \frac{\pi}{6} = 0\right);$$

$$\left[\begin{array}{l}\sin\left(\frac{x}{2} + \frac{\pi}{6}\right) = 0, \\ \cos\left(\frac{3x}{2} - \frac{\pi}{6}\right) = 0\end{array}\right. \quad \left[\begin{array}{l}\frac{x}{2} + \frac{\pi}{6} = \pi n, \\ \frac{3x}{2} - \frac{\pi}{6} = \frac{\pi}{2} + \pi n\end{array}\right. \quad \left[\begin{array}{l}\frac{x}{2} = \pi n - \frac{\pi}{6}, \\ \frac{3x}{2} = \frac{\pi}{2} + \frac{\pi}{6} + \pi n\end{array}\right. \quad \left[\begin{array}{l}x = 2\pi n - \frac{\pi}{3}, \\ x = \frac{4}{9}\pi + \frac{2\pi n}{3}, \ n \in Z.\end{array}\right.$$

$\cos x > 0$, and when $x = 2\pi n + \frac{10\pi}{9}$ $\cos x < 0$.

Answer: $2k\pi - \frac{\pi}{3}$, $\frac{4\pi}{9} + \frac{2\pi n}{3}$ besides $2k\pi + \frac{10\pi}{9}$. $k \in Z$.

An interesting ending has a solution to such an equation:

$\sin^5 x + \cos^5 x = \sec x + \cos ecx.$

Solution:

$\sec x = \frac{1}{\cos x}$; $\cos ecx = \frac{1}{\sin x}$. Then the equation takes the form:

$\sin^5 x + \cos^5 x - \frac{1}{\cos x} - \frac{1}{\sin x} = 0 \Big| \cdot \cos x \cdot \sin x$, because $\sin x \neq 0$ and $\cos x \neq 0$.

$\sin x \cdot \cos x\left(\sin^5 x + \cos^5 x\right) - \left(\sin x + \cos x\right) = 0$ (1)

We transform the expression

$\sin^5 x + \cos^5 x = \left(\sin x + \cos x\right)^5 - A =$ (2) $\left(\sin x + \cos x\right)\left(\sin x + \cos x\right)^4 - A =$

$= \left(\sin x + \cos x\left(\sin x + \cos x\right)^2\right)^2 - A = \left(\sin x + \cos x\right)\left(1 + \sin 2x\right)^2 - A =$ (3)

$= \left(\sin x + \cos x\right)\left(1 + \sin 2x\right)^2 - A = \left(\sin x + \cos x\right)\left(1 + 2\sin 2x + \sin^2 2x\right) = A$

From equality (2) we have:

$A = \left(\sin x + \cos x\right)^5 - \sin^5 x - \cos^5 x = \left(\sin x + \cos x\right)^2 \cdot \left(\sin x + \cos x\right)^3 - \sin^5 x - \cos^5 x =$

$= \left(\sin^2 x + 2\sin x \cos x + \cos^2 x\right) \cdot \left(\sin^3 x + 3\sin^2 x \cdot \cos x + 3\sin x \cdot \cos^2 x + \cos^3 x\right) -$

$- \sin^5 x - \cos^5 x = \sin^5 x + 3\sin^4 x \cdot \cos x + 3\sin^3 x \cdot \cos^2 x + \sin^2 x \cdot \cos^3 x + 2\sin^4 x \cdot \cos x +$

$+ 6\sin^3 x \cos^2 x + 6\sin^2 x \cdot \cos^3 x + 2\sin x \cdot \cos^4 x + \sin^3 x \cos^3 x + 3\sin^2 x \cdot \cos^3 x +$

$+ 3\sin x \cos^4 x + \cos^5 x - \sin^5 x - \cos^5 x = 5\sin^4 x \cdot \cos x + 10\sin^3 x \cos^2 x + 10\sin^2 x \cdot \cos^3 x +$

$+ 5\sin x \cdot \cos^4 x = \left(5\sin^4 x \cos x + 5\sin x \cdot \cos^4 x\right) + \left(10\sin^3 x \cdot \cos^2 x + 10\sin^2 x \cdot \cos^3 x\right) =$

$= 5\sin x \cdot \cos x\left(\sin^3 x + \cos^3 x\right) + 10\sin^2 x \cdot \cos^2 x \cdot \left(\sin x + \cos x\right) = \sin x \cdot \cos x\left(\sin x + \cos x\right) \times$

$\times \left(\sin^2 x - \sin x \cdot \cos x + \cos^2 x\right) + 5 \cdot \sin x \cdot \cos x \cdot 2\sin x \cos x\left(\sin x + \cos x\right) = 5\sin x \cdot \cos x \times$

$\times \left(\sin x + \cos x\right) \cdot \left(\sin^2 - \sin x \cdot \cos x + \cos^2 x + 2\sin x \cdot \cos x\right) = 5 \cdot \frac{1}{2}\sin 2x\left(\sin x + \cos x\right) \times$

$\times \left(1 + \frac{1}{2}\sin 2x\right) = 2{,}5\sin 2x\left(\sin x + \cos x\right) \cdot \left(1 + \frac{1}{2}\sin 2x\right) = \left(\sin x + \cos x\right)\left(2{,}5\sin 2x + 1{,}25\sin^2 2x\right).$

We substitute the expression for A into the equality (3):

$\sin^5 x + \cos^5 x = \left(\sin x + \cos x\right) \cdot \left(1 + 2\sin 2x + \sin^2 2x\right) - \left(\sin x + \cos x\right)\left(2{,}5\sin 2x + 1{,}25\sin^2 2x\right) =$

$= \left(\sin x + \cos x\right)\left(1 + 2\sin 2x + \sin^2 2x - 2{,}5\sin 2x - 1{,}25\sin^2 2x\right) = \left(\sin x \cos x\right) \times$

$\times \left(1 - \frac{1}{2}\sin 2x - \frac{1}{4}\sin^2 2x\right).$

Equation (1) takes the form:

$\sin x \cdot \cos x \cdot \left(\sin x + \cos x\right) \cdot \left(1 - \frac{1}{2}\sin 2x - \frac{1}{4}\sin^2 2x\right) - \left(\sin x + \cos x\right) = 0;$

$\frac{1}{2}\sin 2x \cdot \left(\sin x + \cos x\right) \cdot \left(1 - \frac{1}{2}\sin 2x - \frac{1}{4}\sin^2 2x\right) - \left(\sin x + \cos x\right) = 0;$

$\left(\sin x + \cos x\right) \cdot \left(\frac{1}{2}\sin 2x - \frac{1}{4}\sin^2 2x - \frac{1}{8}\sin^3 2x - 1\right) = 0$

$$\left[\begin{array}{l} \sin x+\cos x=0 \,|: \cos x \qquad tgx=-1; \qquad x=-\frac{\pi}{4}+\pi n \\ \frac{1}{2}\sin 2x-\frac{1}{4}\sin^2 2x-\frac{1}{8}\sin^3 2x-1=0. \end{array}\right.$$

Let the $\sin 2x = y,$ then $|y| \le 1.$

$$\frac{1}{2}y-\frac{1}{4}y^2-\frac{1}{8}y^3-1=0|\cdot(-8)$$

$$y^3+2y^2-4y+8=0.$$

It follows from the properties of the modulus of a number that $|y| \ge y$ and $|y| \ge -y.$

Consider this expression:

$$y^3+2y^2+4y.$$

$$|y|^3+2\cdot|y|^2+4|y| \ge -y^3-2y^2+4y|\times(-1)$$

$$-|y|^3+2|y|^2+4|y| \le y^3+2y^2-4y;$$

$$y^3+2y^2-4y \ge (|y|^3+2|y|^2+4|y|);$$

$$y^3+2y^2-4y \ge -7|+8;$$

$$y^3+2y^2-4y+8 \ge 1.$$

Thus, for $|y| \le 1$ $\quad y^3+2y^2-4y+8 \ne 0,$ ie equation $y^3+2y^2-4y+8=0$ has no roots at most one.

Answer: $-\frac{\pi}{4}+\pi n,\ \ n \in Z.$

A rare equation:

$$\sin\left(\frac{3\pi}{2}\cdot tgx\right)=\cos\left(\frac{3\pi}{2}\cdot ctgx\right).$$

Solution:

$\sin\left(\frac{3\pi}{2}\cdot tgx\right)=\cos\left(\frac{\pi}{2}-\frac{3\pi}{2}\cdot tgx\right);$ Тоді $\cos\left(\frac{\pi}{2}-\frac{3\pi}{2}\cdot tgx\right)-\cos\left(\frac{3\pi}{2}\cdot ctgx\right)=0;$

Convert the difference of trigonometric functions to the product:

$$2\sin\frac{\frac{\pi}{2}-\frac{3\pi}{2}\cdot tgx+3\pi\cdot ctgx}{2}\cdot\sin\frac{\frac{3\pi}{2}ctgx-\frac{\pi}{2}+\frac{3\pi}{2}tgx}{2}=0;$$

$$2\sin\frac{\frac{\pi-3\pi\cdot tgx+3\pi\cdot ctgx}{2}}{2}\cdot\sin\frac{\frac{3\pi\cdot tgx-\pi+3\pi\cdot tgx}{2}}{2}=0;$$

$$2\sin\frac{3\pi ctgx-3\pi\cdot tgx+\pi}{4}\cdot\sin\frac{3\pi ctgx+3\pi\cdot tgx-\pi}{4}=0;$$

This equation is equivalent to the combination of two equations:

$$\left[\begin{array}{l} \sin\frac{3\pi ctgx-3\pi\cdot tgx+\pi}{4}=0, \\ \sin\frac{3\pi ctgx+3\pi\cdot tgx-\pi}{4}=0. \end{array}\right. \quad \left[\begin{array}{l} \frac{3\pi ctgx-3\pi\cdot tgx+\pi}{4}=k\pi, \\ \frac{3\pi ctgx+3\pi\cdot tgx-\pi}{4}=k\pi. \end{array}\right. \quad \left[\begin{array}{l} 3\pi ctgx-3\pi\cdot tgx+\pi=4k\pi, \\ 3\pi ctgx+3\pi\cdot tgx-\pi=4k\pi. \end{array}\right.$$

$$\left[\begin{array}{l}\pi(3ctgx-3tgx+1)=4k\pi|:\pi,\\ \pi(3ctgx+3tgx-1)=4k\pi|:\pi.\end{array}\right. \quad \left[\begin{array}{l}3ctgx-3tgx=4k-1|:3,\\ 3ctgx+3tgx=4k+1|:3.\end{array}\right. \quad \left[\begin{array}{l}ctgx-tgx=\frac{4k-1}{3},\\ ctgx+tgx=\frac{4k+1}{3}.\end{array}\right.$$

$$\left[\begin{array}{l}\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}=\frac{4k-1}{3},\\ \frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}=\frac{4k+1}{3}.\end{array}\right. \quad \left[\begin{array}{l}\frac{\cos^2 x-\sin^2 x}{\sin x\cdot\cos x}=\frac{4k-1}{3},\\ \frac{\cos^2 x+\sin^2 x}{\sin x\cdot\cos x}=\frac{4k+1}{3}.\end{array}\right. \quad \left[\begin{array}{l}\frac{2\cdot\cos 2x}{2\cdot\sin x\cdot\cos x}=\frac{4k-1}{3},\\ \frac{1\cdot 2}{2\cdot\sin x\cdot\cos x}=\frac{4k+1}{3}.\end{array}\right.$$

$$\left[\begin{array}{l}\frac{2\cdot\cos 2x}{\sin 2x}=\frac{4k-1}{3},\\ \frac{2}{\sin 2x}=\frac{4k+1}{3}.\end{array}\right. \quad \left[\begin{array}{l}\frac{\sin 2x}{2\cos 2x}=\frac{3}{4k-1}|\cdot 2,\\ \frac{\sin 2x}{2}=\frac{3}{4k+1}|\cdot 2.\end{array}\right. \quad \left[\begin{array}{l}tg2x=\frac{6}{4k-1},\\ \sin 2x=\frac{6}{4k+1}.\end{array}\right. \quad \left[\begin{array}{l}2x=arctg\frac{6}{4k-1}+\pi k,\\ 2x=(-1)^n\arcsin\frac{6}{4k+1}+\pi k,\end{array}\right.$$

$$\left[\begin{array}{l}x=\frac{1}{2}arctg\frac{6}{4k-1}+\frac{\pi k}{2},\ n\in Z,\\ x=\frac{1}{2}\cdot(-1)^n\arcsin\frac{6}{4k+1}+\frac{\pi k}{2},\ k\in Z.\end{array}\right.$$

Answer: at $n\neq\{-1;0;1\}$, $\frac{1}{2}\arcsin\frac{6}{4k-1}+\frac{\pi k}{2};\ \frac{1}{2}(-1)^n\arcsin\frac{6}{4k-1},\ n\in Z.$

Find the roots of the equation on $\left(\frac{\pi}{4};\frac{3\pi}{4}\right)$ $\sin x-2\sin 2x-\sin 3x=3.$

Solution:

$\sin 2x=2\sin x\cdot\cos x;\quad \sin 3x=4\sin^3+5\sin x;\quad \sin x-4\sin x\cdot\cos x+4\sin^2 x-3\sin x-3=0;$
$\sin x(1-4\cos x+4\sin^2 x-3)-3=0;\quad \sin x(4\sin^2 x-4\cos x-2)-3=0;$
$\sin x(4(1-\cos^2 x)-4\cos x-2)-3=0;\quad \sin x(4-4\cos^2 x-4\cos x-2)-3=0;$
$-\sin x(4\cos^2 x+4\cos x-2)-3=0|\cdot(-1);\quad \sin x(4\cos^2 x+4\cos x-2)+3=0;$
$\sin x(4\cos^2 x+4\cos x+1-1-2)+3=0;\quad \sin x((2\cos x+1)^2-3)+3=0;$
$\sin x(2\cos x+1)^2-3\sin x+3=0;\quad \sin x(2\cos x+1)^2-3(\sin x-1)=0;$
$\sin x(2\cos x+1)^2+3(1-\sin x)=0;\ (1)$

Because the $(2\cos x+1)^2\geq 0$ i $(1-\sin x)\geq 0,$ then equation (1) takes place when

$$\begin{cases}\sin x\cdot(2\cos x+1)^2=0,\\ 1-\sin x=0.\end{cases} \quad \begin{cases}\left[\begin{array}{l}\sin x=0,\\ (2\cos x+1)^2=0\end{array}\right.\\ 1-\sin x=0\end{cases} \quad \begin{cases}\left[\begin{array}{l}x=\pi n,\\ 2\cos x+1=0\end{array}\right.\\ \sin x=1\end{cases} \quad \begin{cases}\left[\begin{array}{l}x=\pi n,\\ \cos x=-\frac{1}{2}\end{array}\right.\\ x=\frac{\pi}{2}+2\pi n\end{cases} \quad \begin{cases}\left[\begin{array}{l}x=\pi n,\\ x=\pm\frac{3}{5}\pi+2k\pi\end{array}\right.\\ x=\frac{\pi}{2}+2\pi n\end{cases}$$

The equations of this system have no common solutions.

Answer: $\varnothing$.

On $(0^0;\ 90^0)$ find the smallest root of the equation $\cos^2 3x+\cos^2 5x=\cos^2 7x+\cos^2 9x.$

Solution:

Let us lower the degree of each of the equations:

$$\frac{1+\cos 6x}{2}+\frac{\cos 10x+1}{2}=\frac{1+\cos 14x}{2}+\frac{1+\cos 8x}{2}|\cdot 2$$

$1+\cos 6x+1+\cos 10x=1+\cos 14x+1+\cos 8x;\quad \cos 6x+\cos 10x=\cos 14x+\cos 18x;$

We transform the sums of cosines into products:

$$2\cos\frac{6x+10x}{2}\cdot\cos\frac{6x-10x}{2}=2\cos\frac{14x+18x}{2}\cdot\cos\frac{14x-18x}{2}\Big|:2;$$

$$\cos 8x\cdot\cos(-2x)=\cos 16x\cdot\cos(-2x),$$

$$\cos 16x\cdot\cos 2x-\cos 8x\cdot\cos 2x=0,$$

$$\cos 2x(\cos 16x-\cos 8x)=0,\quad \cos 2x\cdot 2\cdot\sin\frac{16x+8x}{2}\cdot\sin\frac{16x-8x}{2}=0,$$

$$2\cos 2x\cdot\sin 4x\cdot\sin 12x=0,$$

$$\left[\begin{array}{l}\cos 2x=0,\\ \sin 4x=0,\\ \sin 12x=0\end{array}\right.\quad \left[\begin{array}{l}2x=\dfrac{\pi}{2}+\pi n,\\ 4x=\pi n,\\ 12x=\pi n\end{array}\right.\quad \left[\begin{array}{l}x=\dfrac{90^0}{2}+\dfrac{180^0 n}{2},\\ x=\dfrac{180^0 n}{4},\quad n\in Z\\ x=\dfrac{180^0 n}{12}.\end{array}\right.$$

By condition $\left[\begin{array}{l}0^0<45^0+90^0 n<90^0\big|-45^0,\\ 0^0<45^0 n<90^0,\\ 0^0<15^0 n<90^0\big|:15,\end{array}\right.\quad \left[\begin{array}{l}90n<45\\ n<2\\ n<6\end{array}\right.$

Because the $n\in Z$, then the first inequality of the set of solutions has no.

If $n<2$, then $n<6$. At $n=1$. $x=\dfrac{180\cdot 1}{4}=45^0$; $x=\dfrac{180\cdot 1}{12}=15^0$. $15^0<45^0$.

The smallest root of this equation on $(0^0;90^0)$ $x=15^0$.

Answer: $15^0$.

Find the smallest $x$ in degrees if $-90^0<x<90^0$ i

$$\sin(180^0+x)\cdot\sin(90-7x)=\cos(180^0-3x)\cdot(360^0+5x).$$

Solution:

We apply the reduction formulas:

$$\sin x\cdot\cos 7x=\cos 3x\cdot\sin 5x,\quad \frac{1}{2}\cdot(\sin(x+7x)-\sin(x-7x))=\frac{1}{2}(\sin(3x+5x)-\sin(5x-3x))\big|\cdot 2,$$

$$\sin 8x+\sin 6x=\sin 8x-\sin 2x,\quad \sin 6x+\sin 2x=0,\quad 2\sin\frac{6x+2x}{2}\cdot\cos\frac{6x-2x}{2}=0,$$

$$2\sin 4x\cdot\cos 2x=0$$

$$\left[\begin{array}{l}\sin 4x=0,\\ \cos 2x=0\end{array}\right.\quad \left[\begin{array}{l}4x=\pi n,\\ 2x=\dfrac{\pi}{2}+2\pi n\end{array}\right.\quad \left[\begin{array}{l}x=\dfrac{180n}{4},\\ x=\dfrac{90^0}{2}+180n\end{array}\right.\quad \left[\begin{array}{l}x=45^0\cdot n,\\ x=45+180n\end{array}\right.\quad \left[\begin{array}{l}-90^0<45n<90^0\big|:45^0,\\ -90^0<45^0+180n<90^0\big|-45^0\end{array}\right.$$

$$\left[\begin{array}{l}-2<n<2,\\ -135^0<180n<45^0\end{array}\right.\quad \left[\begin{array}{l}-2<n<2,\\ -\dfrac{3}{4}<n<\dfrac{1}{4}\end{array}\right.\quad \left[\begin{array}{l}n=\{-1;0;1\}\\ n=0\end{array}\right.\quad \begin{array}{l}x=-45^0,\ x=0^0,\ x=45^0\\ x=450+0=450.\end{array}$$

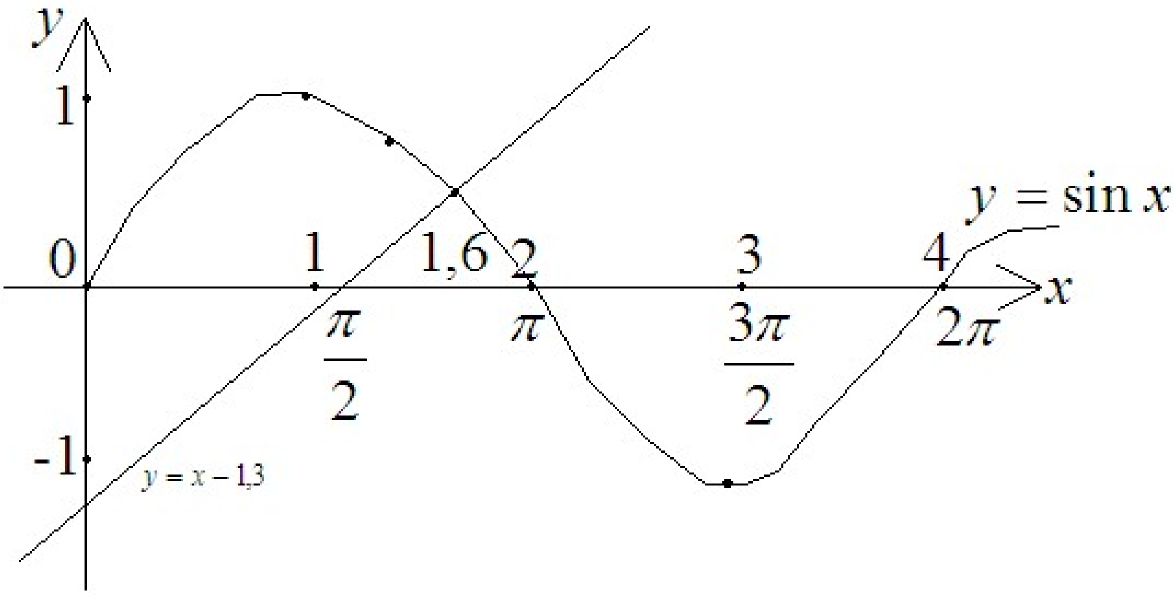
Answer: $-45^0$.

Seductive is the solution of this equation:

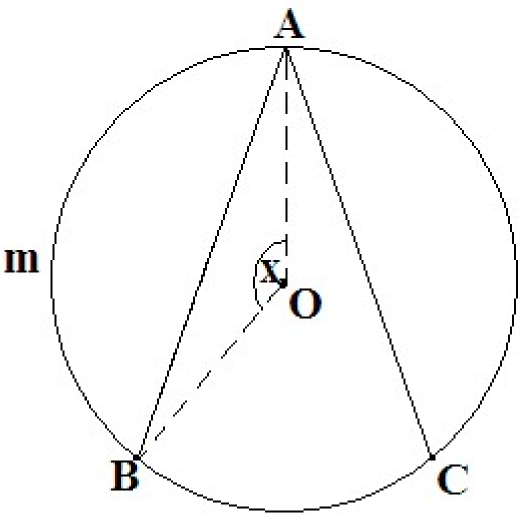
$$\sin x+1{,}3=x.$$

Solution:

$\sin x = x - 1{,}3$. In one coordinate plane, we plot a graph of two functions $y = \sin x$ and $y = x - 1{,}3$




Such equations have a considerable branch of practical application, for example: with two chords extending from one point of the circle, divide the circle into three equal parts.




Solution:

Let the AB and AC – the desired chords, and the central angle $AOB = x$ radian. Then the area of the sector $AmBO = \frac{\pi R^2 x}{2\pi} = \frac{1}{2} R^2 x,$ where $R -$ radius of the circle. Area of a triangle $AOB = \frac{1}{2} \cdot AO \cdot OB \cdot \sin < AOB,$

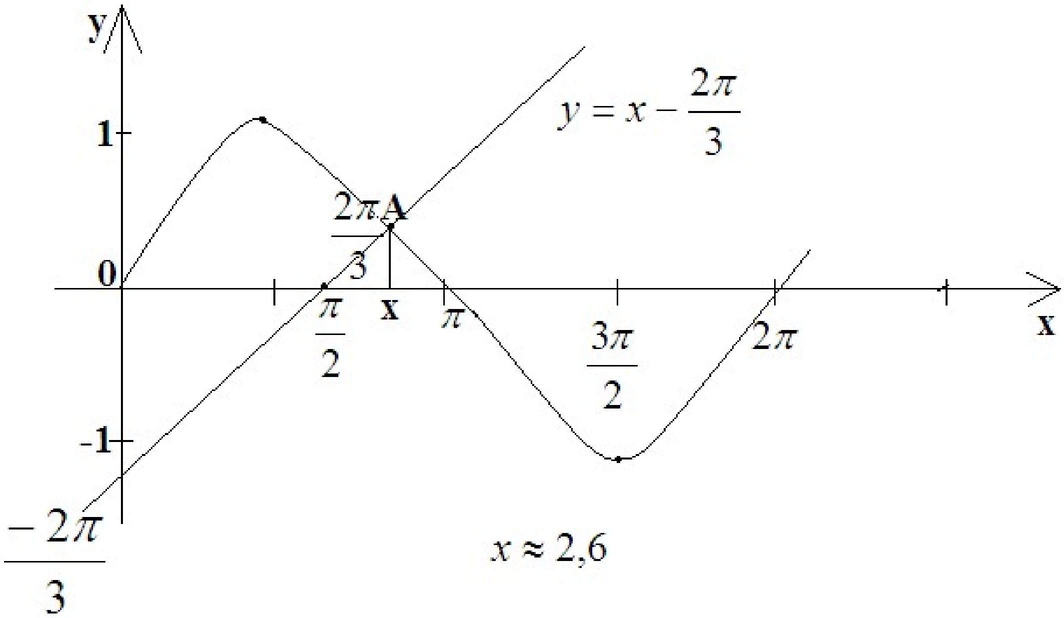
$S_{\Delta AOB} = \frac{1}{2} \cdot R \cdot R \cdot \sin x,\ \ S_{\Delta AOB} = \frac{1}{2} R^2 \sin x.$

We define the segment area $S_{AmB} = \frac{1}{2} R^2 x - \frac{1}{2} R^2 \sin x = \frac{1}{2} R^2 (x - \sin x).$

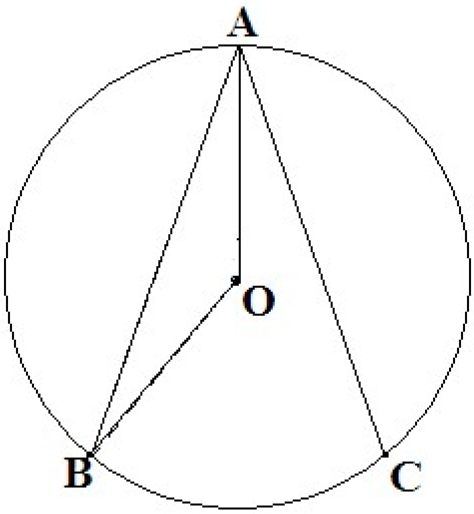
Since, according to the condition of the problem, the area of the sector is one third of the area of the circle, then $\frac{1}{2} R^2 (x - \sin x) = \frac{1}{3} \pi R^2 \Big| : \frac{1}{2} R^2;\ \ x - \frac{2}{3}\pi = \sin x.$

This equation is equivalent to such a system of equations:

$\begin{cases} y = x - \frac{2\pi}{3}, \\ y = \sin x. \end{cases}$ The solution to this system, we obtain a graphical way:




Let us express using "Four-digit math tables" V. M. Bradis $2{,}6 рад \approx 149^0$.




Building a given circle.
Draw a radius OA.
Using the protractor construct central angle $149^0$.
Let's draw a chord AB.
Building the second chord AC = AB.

Answer: In this manner the chord constructed circle divided into three equal parts.

# Self-study assignments:

Solve Equations:

$\sin x = \frac{1}{2}.$ Answer: $(-1)^n \cdot \frac{\pi}{6} + \pi n,\ n \in Z.$

$\cos x = -\frac{1}{2}.$ Answer: $\frac{\pi}{3} + \pi n,\ n \in Z.$

$\cos x = -\frac{1}{2}.$ Answer: $\frac{\pi}{3} + \pi n,\ n \in Z.$

$tgx = \sqrt{3}.$ Answer: $\frac{\pi}{3} + \pi n,\ n \in Z.$

$ctgx = -\frac{\sqrt{3}}{3}.$ Answer: $\frac{2}{3}\pi + \pi n,\ n \in Z.$

$\sqrt{2}\sin x - 1 = 0.$ Answer: $(-1)^n \cdot \frac{\pi}{4} + \pi n,\ n \in Z.$

$2\cos x + \sqrt{3} = 0.$ Answer: $\pm\frac{5\pi}{6} + 2\pi n,\ n \in Z.$

$3\sin x - 4 = 0.$ Answer: $\varnothing.$

$3tgx + 1 = 0.$ Answer: $arctg\left(-\frac{1}{3}\right) + \pi n,\ n \in Z.$

$3ctgx + 2 = 0.$ Answer: $arctg\left(-\frac{2}{3}\right) + \pi n,\ n \in Z.$

$\sin(3x - 1) = \frac{1}{5}.$ Answer: $x = \frac{1}{3}(-1)^n \arcsin\frac{1}{5} + \frac{1}{3} + \frac{\pi n}{3}.$

$\cos\left(x^2 - 2\right) = \frac{1}{2}.$ Answer: $x = \pm\sqrt{2 \pm \frac{\pi}{3} \pm 2\pi n},\ n \in Z.$

$2\sin x - 3\cos x = 0.$ Answer: $x = arctg\frac{3}{x} + \pi n,\ n \in Z.$

$2\sin^2 x + 3\sin x \cdot \cos x + \cos^2 x = 0.$ Answer: $-\frac{\pi}{4} + \pi n,\ -arctg + \pi n,\ n \in Z.$

$2\sin x \cdot \cos x + 5\cos^2 x = 4.$ Answer: $arctg\frac{1+\sqrt{5}}{4} + \pi n,\ arctg\frac{1-\sqrt{5}}{4} + \pi n.$

$\sin^4 x + \cos^4 x = \frac{1}{2}\sin^2 2x.$ Answer: $\frac{\pi}{4} + \frac{\pi n}{2}.$

$\sin^6 x + \cos^6 x = \frac{1}{4}.$ Answer: $\frac{\pi}{4} + \frac{\pi n}{2},\ n \in Z.$

$3\sin x + 4\cos x = 5.$ Answer: $-\arcsin\frac{4}{5} + \frac{\pi}{2} + 2\pi n,\ n \in Z.$

$2 \cdot (\sin x + \cos x) + \sin 2x = -1.$ Answer: $-\frac{\pi}{4} + \pi n,\ n \in Z.$

$\cos 2x + 4\sin^4 x = 8\cos^6 x.$ Answer: $\frac{\pi}{4} + \frac{\pi n}{2},\ n \in Z.$

$2\cos^2 4x + \sin 10x = 1.$ Answer: $-\frac{\pi}{4} + \pi n,\ \frac{\pi}{12} + \frac{\pi n}{9}\ \ n \in Z.$

$\sin^2 x + \cos^2 2x = \sin^4 3x + \cos^2 4x.$ Answer: $\frac{\pi n}{2},\ \frac{\pi}{10} + \frac{\pi n}{5}(n \neq 5l + 2),\ n \in Z.$

$\sin x + ctg\frac{x}{2} = 2.$ Answer: $\frac{\pi}{2} + 2\pi n,\ n \in Z.$

$\sin^2 x + \frac{1}{\sin^2 x} = \sin x - \frac{1}{\sin x} + \frac{7}{4}.$ Answer: $(-1)^{n+1}\arcsin\frac{\sqrt{17}-1}{4} + \pi n,\ n \in Z.$

$\sin\left(\frac{3\pi}{5}+x\right)=2\sin\left(\frac{\pi}{5}-\frac{x}{2}\right).$ Answer: $\pi n,\ \frac{\pi}{5}-\frac{x}{2}+\pi n,\ n\in Z.$

$4\cos^2 x+4\sin x-1=0.$ Answer: $(-1)^{n+1}\cdot\frac{\pi}{6}+\pi n,\ n\in Z.$

$\sqrt{3}\cos^2 3x+\sin 6x-\sqrt{3}\sin^2 3x=0.$ Answer: $-\frac{\pi}{18}+\frac{\pi n}{6},\ n\in Z.$

$\frac{\cos 2x}{1+\sin 2x}=0.$ Answer: $\frac{\pi}{4}+\frac{\pi n}{2},\ n\in Z.$

$1-\cos x-2\sin\frac{x}{2}=0.$ Answer: $2\pi n;\ \pi+4\pi n,\ n\in Z.$

$2\sin x\cdot\cos 2x-1+2\cos 2x-\sin x=0.$ Answer: $\pm\frac{\pi}{6}+\pi n,\ \ -\frac{\pi}{2}+2\pi n,\ \ n\in Z.$

$\frac{\cos 3x}{\sin 2x}=\frac{\cos 5x}{\sin 2x}.$ Answer: $\frac{\pi}{4}+\frac{\pi n}{2},\ n\in Z.$

$\cos^3 x+\sin^3 x=\cos 2x.$ Answer: $-\frac{\pi}{4}+\pi n,\ 2\pi n,\ -\frac{\pi}{2}+2\pi n,\ n\in Z.$

$\cos 3x+\sin 5x=0.$ Answer: $\frac{3\pi}{4}+\pi n,\ \frac{3}{16}\pi+\frac{\pi n}{4},\ n\in Z.$

$\sin x\cdot\cos 5x=\sin 9x\cdot\cos 3x.$ Answer: $\frac{\pi n}{8},\ n\in Z.$

$\sin 2x+\sin 3x+\sin 4x=3.$ Answer: $\varnothing.$

$\cos^2 2x+\frac{1}{4}\sin^2 4x+1=\sin 4x\cdot\cos 2x+\sin^2 x.$ Answer: $\varnothing.$

$2\sin^2 x+\sin x-1=0.$ Answer: $(-1)^n\cdot\frac{\pi}{6}+\pi n;\ -\frac{\pi}{2}+2\pi n,\ n\in Z.$

$5\sin^2 x-3\sin x\cdot\cos x-2\cos^2 x=0.$ Answer: $\frac{\pi}{4}+\pi n,\ arctg\left(-\frac{2}{5}\right)+\pi n,\ n\in Z.$

$2\sin^2 x-5\sin x\cdot\cos x-8\cos^2 x=-2.$ Answer: $arctg 2+\pi n,\ arctg\left(-\frac{3}{4}\right)+\pi n,\ n\in Z.$

$2\sin^2 x+\cos x-1=0.$ Answer: $2\pi n,\ \pm\frac{2\pi}{3}+2\pi n,\ n\in Z.$

$4\sin^2 x-4\cos x-1=0.$ Answer: $\pm\frac{\pi}{3}+2\pi n,\ n\in Z.$

$12\cos^2 x+\sin x-11=0.$ Answer: $(-1)^n\arcsin\frac{1}{3}+\pi n,\ (-1)^{n+1}\arcsin\frac{1}{4}+\pi n,\ n\in Z.$

$6\cos^2 x++\sin x-2=0.$ Answer: $(-1)^{n+1}\frac{\pi}{6}+\pi n,\ n\in Z.$

$\cos 2x+\cos^2 x=0.$ Answer: $\pm\frac{1}{2}\arccos\left(-\frac{1}{3}\right)+\pi n,\ n\in Z.$

$\sin 3x=\sin x.$ Answer: $\pi n,\ \pm\frac{\pi}{4}+\pi n,\ n\in Z.$

$\sin 2x=\cos^2 x.$ Answer: $\frac{\pi}{2}(2\pi+1),\ arctg 2+\pi n,\ n\in Z.$

$\sin x\cdot\cos x\cdot\cos 2x=\frac{1}{8}.$ Answer: $(-1)^n\cdot\frac{\pi}{24}+\frac{\pi n}{4},\ n\in Z.$

$4\sqrt{3}\cos x - 2\sin x = 5.$ Answer: $2arctg\left(2-\sqrt{3}\right)+2\pi n,\ 2\text{arctg}\,\frac{-26+7\sqrt{3}}{23}+2\pi n,\ n\in Z.$

$2\sin x - 2\cos x = 1-\sqrt{3}.$ Answer: $\left(-1\right)^n \arcsin\frac{1-\sqrt{3}}{2\sqrt{2}}+\frac{\pi}{4}+\pi n,\ n\in Z.$

$\left(1+\cos 4x\right)\cdot\sin 2x = \cos^2 2x$ на $\left(180^\circ;225^\circ\right).$ Answer: $195^\circ.$

$\frac{\sin^2 x - 4\sin^2 x}{\sin^2 2x + 4\sin^2 x - 4}+1 = 2tg^2 x$ на $\left(90^\circ;180^\circ\right).$ Answer: $135^\circ.$

$\sin 2x = \cos^4\frac{x}{2}-\sin^4\frac{x}{2}$ на $\left(90^\circ;\ 180^\circ\right).$ Answer: $150^\circ.$

$\left(\sin^2 2x - 4\sin^2 x\right)\cdot\left(\sin^2 2x + 4\sin^2 x - 4\right)^{-1} = 2tg^2 x$ на $\left(0^\circ;\ 90^\circ\right).$ Answer: $45^\circ.$

$\sin 5x - \sin x - \cos 3x = 0$ на $\left(0^\circ;\ 30^\circ\right).$ Answer: $15^\circ.$

How many roots does the equation have $\cos x - \cos 3x - \sin 2x = 0$ on $\left(0;\ \pi\right].$
Answer: 5.

$\sin^4 2x + \cos^4 2x = \sin 2x\cdot\cos 2x.$ Answer: $\frac{\pi}{16}+\frac{\pi n}{8},\ n\in Z.$

$$3\sin^2\frac{x}{2}\cdot\cos\left(\frac{3\pi}{2}+\frac{x}{2}\right)+3\sin^2\frac{x}{2}-\sin\frac{x}{2}\cdot\cos^2\frac{x}{2}-\sin^2\left(\frac{\pi}{2}+x\right)\cdot\cos\frac{x}{2}=0.$$

Answer: $-\frac{\pi}{2}+2\pi n,\ \pm\frac{\pi}{3}+2\pi k,\ n\in Z,\ k\in Z.$

$\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = x^2-4x+5.$ Answer: $\varnothing.$

$\frac{2tg\frac{3}{2}x}{1+tg^2\frac{3}{2}x} = 2x^2-8x+9.$ Answer: $\varnothing.$

$\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = 2x^2-4x+3.$ Answer: $\varnothing.$

$\cos x\cdot\cos\left(2x-2\pi\right)-\cos\left(\frac{3}{2}\pi-2x\right)\cos\left(\frac{\pi}{2}+x\right) = 2x^2-12x+19.$ Answer: $\varnothing.$