Section 24
Derivative of a function

The derivative of a function is the limit of the ratio of the increment of the function to the increment of the argument when the latter tends to zero.

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

The operation of finding the derivative of a function is called differentiation of the function.

Differentiation rules:

1). $c'=0,\ c-$ constant;

2). $x'=1,\ x-$ variable

2a). $\left(\frac{1}{x}\right)'=-\frac{1}{x^2};$

2б). $\left(\frac{a}{x}\right)'=-\frac{a}{x^2};$

3). $(cu)'=c\cdot u',\ c-$ constant, $u-$ variable.

4). $\left(u\pm v'\right)=u'\pm v';$

5). $\left(u\cdot v'\right)=u'\cdot v+u\cdot v';$

6). $\left(\frac{u}{v}\right)'=\frac{u'\cdot v-u\cdot v'}{v^2}.$

Table of derivatives:

1). $\left(x^n\right)'=n\cdot x^{n-1};$

2). $\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}};$

3). $\left(a^x\right)'=a^x\cdot\ln a,\ \ (a>0, a\neq 1);$

4). $\left(e^x\right)'=e^x;$

5). $\left(\log_a x\right)'=\frac{1}{\ln a}\cdot\frac{1}{x};$

6). $(\ln x)'=\frac{1}{x};$

7). $(\sin x)'=\cos x;$

8). $(\cos x)'=-\sin x;$

9). $(tgx)'=\frac{1}{\cos^2 x};$

10). $(ctgx)'=-\frac{1}{\sin^2 x}.$

Derivative of a complex function $f(\varphi(\psi(x)))' = f' \cdot \varphi' \cdot \psi'$.

Exercises:

$$(x^4)' = 4x^3; \quad (x^5)' = 5x^4; \quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}; \; (\sqrt[4]{x^3})' = \left(x^{\frac{3}{4}}\right)' = \frac{3}{4}x^{-\frac{1}{4}} = \frac{3}{4}\cdot\frac{1}{x^{\frac{1}{4}}} = \frac{3}{4\sqrt[4]{x}}.$$

$$(5x^3)' = 5\cdot 3x^2 = 15x^2;$$

$$(-4x^2)' = -4\cdot 2x = -8x;$$

$$(7\sqrt{x})' = 7\cdot\frac{1}{2\sqrt{x}}; \; \left(\frac{8}{x^2}\right)' = (8\cdot x^{-2})' = 8\cdot(-2)x^{-3} = -\frac{16}{x^3};$$

$$(4\sqrt[3]{x^2})' = \left(4x^{\frac{2}{3}}\right)' = 4\cdot\frac{2}{3}\cdot x^{-\frac{1}{3}} = \frac{8}{3\sqrt[3]{x}};$$

$$\left(\frac{6}{\sqrt{x}}\right)' = 6\cdot\left(\frac{1}{\sqrt{x}}\right)' = 6\cdot\left(-\frac{1}{x}\right)\cdot(\sqrt{x})' = -\frac{6}{x}\cdot\frac{1}{2\sqrt{x}} = -\frac{3}{x\sqrt{x}};$$

$$\left(\frac{4}{\sqrt[3]{x^2}}\right)' = \left(\frac{4}{x^{\frac{2}{3}}}\right)' = 4\cdot\left(\frac{1}{x^{\frac{2}{3}}}\right)' = 4\cdot\left(-\frac{1}{x^{\frac{4}{3}}}\right)\cdot\frac{2}{3}x^{-\frac{1}{3}} = -\frac{8}{3x^{\frac{5}{3}}} = -\frac{8}{3x\sqrt[3]{x^2}};$$

$$\left(-\frac{5}{4x^3}\right)' = -\frac{5}{4}\left(-\frac{1}{x^6}\right)\cdot 3x^2 = \frac{15}{4\cdot x^4};$$

$$\left(\frac{4\sqrt{x}}{8}\right)' = \frac{7}{8}\cdot\frac{1}{2\sqrt{x}} = \frac{7}{16\sqrt{x}};$$

$$(5x^3 - 3x^2 + x - 1)' = 15x^2 - 6x + 1;$$

$$\left((5x^2+7)^3\right)' = 3\cdot(5x+7)^2\cdot(10x) = 30x(5x+7)^2;$$

$$\left((1+5x-8x^2)^5\right)' = 5(1+5x-8x^2)^4\cdot(5-16x);$$

$$\left(\left(1+2\sqrt{x}-\frac{3}{x^2}\right)^4\right)' = 4\left(1+2\sqrt{x}-\frac{3}{x^2}\right)\cdot\left(\frac{1}{\sqrt{x}}+\frac{6}{x^3}\right);$$

$$\left(\sqrt{x^2+2}\right)' = \frac{1}{2\sqrt{x^2+2}}\cdot 2x = \frac{x}{\sqrt{x^2+2}};$$

$$(\sqrt{3})' = \frac{1}{2\sqrt{3x}}\cdot 3 = \frac{3}{2\sqrt{3x}};$$

$$\left(\frac{2}{(3x^2-5)^3}\right)' = -\frac{2}{(3x^2-5)^6}\cdot 3(3x^2-5)^2\cdot 6x = -\frac{36x(3x^2-5)^2}{(3x^2-5)^6} = -\frac{36x}{(3x^2-5)^4};$$

$$\left(\frac{1}{\sqrt{x^2+x+1}}\right)' = -\frac{1}{2\sqrt{x^2+x+1}}\cdot\frac{1}{2\sqrt{x^2+x+1}}\cdot(2x+1) = -\frac{2x+1}{2(x^2+x+1)\cdot\sqrt{x^2+x+1}};$$

$$\left(\sqrt[3]{\left(3t-2t^2\right)}\right)'=\left(\left(3t-2t^2\right)^{\frac{1}{3}}\right)'=\frac{1}{3}\cdot\left(3t-2t^2\right)^{-\frac{2}{3}}\cdot(3-4t)=\frac{3-4t}{3\sqrt{\left(3t-2t^2\right)^3}};$$

$$\left(\sqrt[4]{\left(2t^2-t^3\right)^3}\right)'=\left(\left(2t^2-t^3\right)^{\frac{3}{4}}\right)'=\frac{3}{4}\left(2t^2-t^3\right)^{-\frac{1}{4}}\cdot\left(4t-3t^2\right)=\frac{3\cdot\left(4t-3t^2\right)}{4\cdot\sqrt[4]{2t^2-t^3}};$$

$$\left(x^2\cdot(5x-4)^6\right)'=2x\cdot(5x-4)^6+x^2\cdot 6(5x-4)^5\cdot 5=2x(5x-4)^5\cdot(5x-4+15x)=2x(5x-4)^5(20x-4)=$$
$$=8x(5x-4)^5(5x-1);$$

$$\left(\left(40-12x+\frac{27}{5}x^2\right)\cdot\sqrt{5+3x}\right)'=\left(-12+\frac{54}{5}x\right)\cdot\sqrt{5+3x}+\left(40-12x+\frac{27}{5}x^2\right)\cdot\frac{3}{2\sqrt{5+3x}}=$$

$$=\left(-12+\frac{54}{5}x\right)\sqrt{5+3x}+\frac{\frac{200-60x+27x^2}{5}}{2\sqrt{5+3x}}\cdot 3=\frac{-60+54x}{5}\cdot\sqrt{5+3x}+\frac{\frac{600-180x+81x^2}{5}}{2\sqrt{5+3x}}=$$

$$=\frac{\frac{-120+108x}{5}\cdot(5+3x)+\frac{600-180x+81x^2}{5}}{2\sqrt{5+3x}}=\frac{\frac{-600-360x+540x+324x^2+600-180x+81x^2}{5}}{2\sqrt{5+3x}}=$$

$$=\frac{81x^2}{2\sqrt{5+3x}};$$

$$\left(\left(\frac{2}{27x}-\frac{1}{9x^2}\right)\cdot\sqrt{3x+x^2}\right)'=\left(-\frac{2}{27x^2}+\frac{1\cdot 2x}{9x^4}\right)\sqrt{3x+x^2}+\frac{1}{2\sqrt{3x+x^2}}\cdot(3+2x)\cdot\left(\frac{2}{27x}-\frac{1}{9x^2}\right)=$$

$$=\frac{-2x^2+6x}{27x^4}\cdot\sqrt{3x+x^2}+\frac{3+2x}{2\sqrt{3x+x^2}}\cdot\frac{2x-3}{27x^2}=\frac{\left(-2x^2+6x\right)\cdot\sqrt{3x+x^2}}{27x^4}+\frac{4x^2-9}{2\cdot 27x^2\cdot\sqrt{3x+x^2}}=$$

$$=\frac{2\left(-2x^2+6x\right)\cdot\left(3x+x^2\right)+\left(4x^2-9\right)\cdot x^2}{2\cdot 27x^4\cdot\sqrt{3x+x^2}}=\frac{\left(-4x^2+12x\right)\cdot\left(3x+x^2\right)+4x^4-9x^2}{54x^4\cdot\sqrt{3x+x^2}}=$$

$$=\frac{-12x^3-4x^4+36x^2+12x^3+4x^4-9x^2}{54x^4\cdot\sqrt{3x+x^2}}=\frac{27x^2}{54x^4\cdot\sqrt{3x+x^2}}=\frac{1}{2x^2\cdot\sqrt{3x+x^2}};$$

$$\left(\left(8x^3-21\right)\cdot\sqrt[3]{\left(7+4x^3\right)^2}\right)'=24x^2\cdot\sqrt[3]{\left(7+4x^3\right)^2}+\left(8x^3-21\right)\cdot\frac{2}{3}\left(7+4x^3\right)^{-\frac{1}{3}}\cdot 12x^2=$$

$$=24x^2\cdot\sqrt[3]{\left(7+4x^3\right)^2}+\frac{8x^2\cdot\left(8x^3-21\right)}{\sqrt[3]{7+4x^3}}=\frac{24x^2\sqrt[3]{\left(7+4x^3\right)^2}+8x^2\left(8x^3-21\right)}{\sqrt[3]{7+4x^3}}=$$

$$=\frac{24x^2\left(7+4x^3\right)+8x^2\cdot\left(8x^3-21\right)}{\sqrt[3]{7+4x^3}}=\frac{168x^2+96x^5+64x^5-168x^2}{\sqrt[3]{7+4x^3}}=\frac{160x^5}{\sqrt[3]{7+4x^3}}.$$

$$\left((4x-7)\cdot(3x+7)\cdot\sqrt[3]{3x+7}\right)'=\left(\left(12x^2+28x-21x-49\right)\cdot\sqrt[3]{3x+7}\right)'=\left(\left(12x^2+7x-49\right)\cdot\sqrt[3]{3x+7}\right)'=$$

$$=(24x+7)\cdot\sqrt[3]{3x+7}+\left(12x^2+7x-49\right)\cdot\frac{(3x+7)^{-\frac{2}{3}}\cdot 3}{-3}$$

(see after the rule)

To calculate the derivative of the product of three or more functions, it is appropriate to use the following rule:
1) determine the derivative of the first factor;
2) multiply it by the product of all other factors;
3) add the derivative of the second factor multiplied by the product of the other two factors;
4) add the derivative of the third factor multiplied by the product of the other two factors.
That is, if f, φ and $g-$ differentiated functions, then

$$(f\cdot\varphi\cdot g)' = f'\cdot\varphi\cdot g + f\cdot\varphi'\cdot g + f\cdot\varphi\cdot g'.$$

$$\left((4x-7)\cdot(3x+7)\cdot\sqrt[3]{3x+7}\right)' = (4x-7)'\cdot(3x+7)\cdot\sqrt[3]{3x+7} + (4x-7)\cdot(3x+7)'\cdot\sqrt[3]{3x+7} +$$

$$+(4x-7)\cdot(3x+7)\cdot\left(\sqrt[3]{3x+7}\right)' = 4\cdot(3x+7)\cdot\sqrt[3]{3x+7} + 3\cdot(4x-7)\cdot\sqrt[3]{3x+7} +$$

$$+(4x-7)\cdot(3x+7)\cdot\frac{1\cdot 3}{3\cdot(3x+7)^{\frac{2}{3}}} = 4\cdot(3x+7)\cdot\sqrt[3]{3x+7} + 3\cdot(4x-7)\cdot\sqrt[3]{3x+7} + (4x-7)\cdot\sqrt[3]{3x+7} =$$

$$= \sqrt[3]{3x+7}\cdot(12x+28+12x-21+4x-7) = \sqrt[3]{3x+7}\cdot 28x = 28x\cdot\sqrt[3]{3x+7}.$$

$$\left(\frac{5+3x+x^2}{5-3x+x^2}\right)' = \frac{(3+2x)\cdot(5-3x+x^2)-(5+3x+x^2)\cdot(-3+2x)}{(5-3x+x^2)^2} =$$

$$= \frac{15-9x+3x^2+10x-6x^2+2x^3+15-10x+9x-6x^2+3x^2-2x^3}{(5-3x+x^2)^2} =$$

$$= \frac{30+3x-6x^2}{(5-3x+x^2)^2} = \frac{6\cdot(5-x^2)}{(5-3x+x^2)^2}.$$

$$\left(\frac{x}{\sqrt{1+x^2}}\right)' = \frac{1\cdot\sqrt{1+x^2} - x\cdot\frac{1\cdot 2x}{2\cdot\sqrt{1+x^2}}}{1+x^2} = \frac{\frac{2(1+x^2)-2x^2}{2\cdot\sqrt{1+x^2}}}{1+x^2} = \frac{2+2x^2-2x^2}{2\cdot\sqrt{1+x^2}\cdot(1+x^2)} = \frac{1}{\sqrt{(1+x^2)^3}}.$$

$$(\sin kx)' = \cos kx\cdot(kx)' = k\cdot\cos kx.$$

$$(\sin kx)' = \cos kx\cdot(kx)' = k\cdot\cos x.$$

$$(tgkx)' = \frac{1}{\cos^2 kx}\cdot(kx)' = \frac{k}{\cos^2 kx}.$$

$$(ctgkx)' = \frac{1}{\cos^2 kx}\cdot(kx) = \frac{k}{\cos^2 kx}.$$

$$(\sin 2x^2)' = \cos 2x^2\cdot(2x^2)' = \cos 2x^2\cdot 4x = 4x\cdot\cos 2x^2.$$

$$(\sin\sqrt{x})' = \cos\sqrt{x}\cdot(\sqrt{x})' = \frac{1}{2\sqrt{x}}\cdot\cos\sqrt{x}.$$

$$\left(tg\frac{1+x}{x}\right)' = \frac{1}{\cos^2\frac{1+x}{x}}\cdot\left(\frac{1+x}{x}\right)' = \frac{1}{\cos^2\frac{1+x}{x}}\cdot\left(\frac{1}{x}+1\right)' = -\frac{1}{x^2}\cdot\frac{1}{\cos^2\frac{1+x}{x}}.$$

$$\left(\cos\sqrt{\frac{1}{1+x}}\right)' = -\sin\sqrt{\frac{1}{1+x}}\cdot\frac{1}{2\cdot\sqrt{\frac{1}{1+x}}}\cdot\left(-\frac{1}{\left(\frac{1}{1+x}\right)^2}\right) = \frac{\sin\sqrt{\frac{1}{1+x}}}{2\cdot\left(\frac{1}{1+x}\right)^{2,5}}.$$

$$\left(3\sin^2 x\right)' = 3\cdot 2\sin x\cdot\cos x = 3\cdot\sin 2x.$$

$$\left(\cos^6 x\right)' = 6\cos^5 x\cdot(-\sin x) = -3\sin 2x\cdot\cos 4x.$$

$$\left(\sqrt{\sin x}\right)' = \frac{1}{2\sqrt{\sin x}}\cdot\cos x.$$

$$\left(\sqrt{\sin^2 x+\cos^3 4x}\right)' = \frac{1\cdot\left(2\sin x\cdot\cos x+3\cdot 3\cos^2 4x\cdot(-\sin 4x)\cdot 4\right)}{2\cdot\sqrt{\sin^2 x+\cos^3 4x}} = \frac{\sin 2x-18\sin 8x\cdot\cos 4x}{2\cdot\sqrt{\sin^2 x+\cos^3 4x}}.$$

$$\left(\frac{1}{\cos^3 x}\right)' = -\frac{1}{\cos^6 x}\cdot 3\cdot\cos^2 x\cdot(-\sin x) = -\frac{3\sin x}{\cos^4 x}.$$

$$\left(x^3\cdot\sin x+3x^2\cos x-6x\cdot\sin x-6\cos x\right)' =$$

$$= 3x^2\cdot\sin x+x^3\cdot\cos x+6x\cdot\cos x+3x^2\cdot(-\sin x)-6\cdot\sin x-6x\cos x-6\cdot(-\sin x) = x^3\cdot\cos x.$$

In cases of uncertainties of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ you can use the rule of L'Hôpital (French mathematician of the 18th century), which consists in the fact that the calculation of the limit of the ratio is replaced by the calculation of the limit of the ratio of the derivatives of the numerator and denominator.

$$\lim_{x\to a}\frac{x^m-a^m}{x^n-a^n} = \lim_{x\to a}\frac{\left(x^m-a^m\right)'}{\left(x^n-a^n\right)'} = \lim_{x\to a}\frac{m\cdot x^{m-1}-0}{n\cdot x^{n-1}-0} = \frac{m\cdot x^{m-1}}{n\cdot x^{n-1}} = \frac{m}{n}\cdot a^{m-1-(n-1)} = \frac{m}{n}a^{m-n}.$$

$$\lim_{x\to -1}\frac{x^3-5x^2+2x+8}{x^4-2x^3-16x^2+2x+15} = \lim_{x\to -1}\frac{3x^2-10x+2}{4x^3-6x^2-32x+2} =$$

$$= \frac{3\cdot(-1)^2-10\cdot(-1)+2}{4\cdot(-1)^3-6\cdot(-1)^2-32\cdot(-1)+2} = \frac{3+10+2}{-4-6+32+2} = \frac{15}{24} = \frac{5}{8}.$$

If after the first derivatives of the numerator and denominator, an uncertainty of the form is again formed $\frac{0}{0}$ or $\frac{\infty}{\infty}$, you need to move on to other derivatives, etc.

Self-study assignments:

$\left(7x^6\right)'$. Answer: $42x^5$.

$\left(8\sqrt{x}\right)'$. Answer: $\frac{4}{\sqrt{x}}$.

$\left(\frac{4}{x^5}\right)'$. Answer: $-\frac{20}{x^6}$.

$\left(\frac{5}{\sqrt[4]{x^3}}\right)'$. Answer: $-\frac{15}{4x\cdot\sqrt[4]{x^3}}$.

$\left(\frac{\sqrt[6]{x^5}}{8}\right)'$. Answer: $\frac{5}{48\cdot\sqrt[6]{x}}$.

$\left(\frac{4\sqrt{x}}{7}\right)'$. Answer: $\frac{2}{7\sqrt{x}}$.

$\left(\frac{1}{3}x^3-\frac{3}{2}x^4+\frac{13}{5}x^5-2x^6+\frac{4}{7}x^7\right)'$. Answer: $x^2\cdot(1-6x+13x^2-12x^3+4x^4)$.

$\left(9x^7-\frac{3}{x^5}+\frac{3}{x^{11}}\right)'$. Answer: $63^6+\frac{15}{x^6}+\frac{33}{x^{12}}$.

$\left(3x^2\sqrt[3]{x}-4x\sqrt[4]{x^3}+9\sqrt[3]{x^2}-6\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{4}{7x^2\cdot\sqrt[3]{x}}\right)'$.

Answer: $7x\sqrt[3]{x}-7\sqrt[4]{x}+\frac{6}{\sqrt[3]{x}}-\frac{3}{\sqrt{x}}-\frac{2}{x\sqrt{x}}+\frac{4}{3x^3\sqrt[3]{x}}$.

$\left(\frac{5x^2}{\sqrt[3]{x^2}}+30\cdot\sqrt[15]{x}+\frac{6}{\sqrt[3]{x}}\right)'$. Answer: $8\cdot\sqrt[5]{x^3}+\frac{\sqrt[15]{x}}{x}-\frac{2}{x\cdot\sqrt[3]{x}}$.

$\left(27x^3-\frac{81}{2}x^2\cdot\sqrt[3]{x^2}+12x^2+\frac{12}{5}x\cdot\sqrt[3]{x^2}\right)'$. Answer: $\left(9x-6\cdot\sqrt[3]{x^2}-2\cdot\sqrt[3]{x}\right)^2$.

$\left((5x^2+7x+2)^3\right)'$. Answer: $3\cdot(5x^2+7x+2)^2\cdot(10x+7)$.

$\left((5x^3+4x^2+8)^4\right)'$. Answer: $4\cdot(5x^3+4x^2+8)^3\cdot(15x^2+8x)$.

$\left(\sqrt{3x^2+5x+1}\right)'$. Answer: $\frac{6x+5}{2\sqrt{3x^2+5x+1}}$.

$\left(\frac{1}{\sqrt{x^2+5}}\right)'$. Answer: $-\frac{x}{(x^2+5)\cdot\sqrt{x^2+5}}$.

$\left(\frac{10}{(4x^3-5x^2+7x+1)^2}\right)'$. Answer: $-\frac{40\cdot(12x^2-10x+7)}{(4x^3-5x^2+7x-1)^5}$.

$\left(\sqrt[3]{4+2\sqrt{3x}+3x}\right)'$. Answer: $\frac{1+\sqrt{3x}}{\sqrt{3x}\cdot\left(\sqrt[3]{4+2\sqrt{3x}+3x}\right)^2}$.

$\left(\sqrt[4]{\left(3+4\cdot\sqrt[3]{2x}\right)^3}\right)'$. Answer: $\frac{\sqrt[3]{2}}{\sqrt[3]{x^2}\cdot\sqrt[4]{3+4\cdot\sqrt[3]{2x}}}$.

$\left((5x^2-7x+2)\cdot(15x^2+5)^3\right)'$. Answer:
$(10x-7)\cdot(15x^2+5)^3+90x\cdot(15x^2+5)^2\cdot(5x^2-7x+2)$.

$\left(\left(\frac{10}{3}-2x+x^2\right)\cdot\sqrt{(5+2x)^3}\right)'$. Answer: $7x^2\cdot\sqrt{5+2x}$.

$\left(\left(3x^4+4\right)\cdot\sqrt[4]{9x^4-3}\right)'$. Answer: $\dfrac{135x^7}{\sqrt[4]{\left(9x^4-3\right)^3}}$.

$\left(\left(\dfrac{2}{3x^3}+\dfrac{28}{27x}\right)\cdot\sqrt{7x^2-9}\right)'$. Answer: $\dfrac{18}{x^4\cdot\sqrt{7x^2-9}}$.

$\left(\dfrac{x}{1+x^2}\right)'$. Answer: $\dfrac{1-x^2}{\left(1+x^2\right)^2}$.

$\left(\dfrac{1+x^2}{1-x^2}\right)'$. Answer: $\dfrac{4x}{\left(1-x^2\right)^2}$.

$(\sin 3x)'$. Answer: $3\cos x$.

$(\sin 5x)'$. Answer: $5\cos 5x$.

$(\sin 15x)'$. Answer: $15\cos 15x$.

$(\sin 4x)'$. Answer: $-4\sin 4x$.

$(-\cos 3x)'$. Answer: $3\sin 3x$.

$(\cos 9x)'$. Answer: $-9\sin 9x$.

$\left(\sin\sqrt{\dfrac{1}{1-x}}\right)'$. Answer: $\dfrac{\cos\sqrt{\dfrac{1}{1-x}}}{2\cdot\left(\dfrac{1}{1-x}\right)^{2,5}}$.

$\left(\sqrt{\sin x}\right)'$. Answer: $\dfrac{\cos x}{2\sqrt{\sin x}}$.

$\left(\sqrt{\dfrac{1}{\cos x}}\right)'$. Answer: $\dfrac{1}{2\sqrt{\dfrac{1}{\cos x}}}\cdot\dfrac{tgx}{\cos x}$.

$\left(\dfrac{1-\sin x}{1+\sin x}\right)'$. Answer: $-\dfrac{2\cos x}{\left(-1+\sin x\right)^2}$.

$(\sin 3x)'$. Answer: $3\sin^2 x\cdot\cos x$.

$\left(tg^3x\right)'$. Answer: $3tg^2x\cdot\dfrac{1}{\cos^2 x}$.

$\left(ctgx^4x\right)'$. Answer: $-4ctg^3x\cdot\dfrac{1}{\sin^2 x}$.

$\left(5\cos^5 x\right)'$. Answer: $-24\cos^4 x\cdot\sin x$.

$(7\cdot tg6x)'$. Answer: $42tg^5x\cdot\dfrac{1}{\cos^2 x}$.

$\left(8\sin^2 x\right)'$. Answer: $8\sin 2x$.

$\left(tgx+\dfrac{1}{3}tg^3x\right)'$. Answer: $\dfrac{1}{\cos^4 x}$.

$\left(\left(\frac{2}{\cos^4 x}+\frac{3}{\cos^2 x}\right)\cdot\sin x\right)'$. Answer: $\frac{8-3\cos^4 x}{\cos^5 x}$.

$(tgx-ctgx-2)'$. Answer: tg^2x+ctg^2x.

$\left(\left(\cos^2 x+\frac{2}{3}\right)\cdot\sin^3 x\right)'$. Answer: $5\sin^2 x\cdot\cos^3 x$.

$\lim\limits_{x\to 5}\frac{x^3-8x^2+17x-10}{x^4-5x^3-2x^2+11x-5}$. Answer: $\frac{3}{29}$.

$f(x)=\frac{x^3-2}{x^3+2}$; $f'(1)-?$ Answer: $1\frac{1}{3}$.

$f(x)=\cos^2 x-\sin^2 x$; $f''\left(\frac{\pi}{2}\right)-?$ Answer: 0.

$f(x)=x^3\ln x-\sin^4 x$; $f'''(x)-?$ Answer: $6\ln x+64\cos 4x+11$.

$S=-\frac{1}{6}t^3+3\cdot t^2-5,\ a=0,\ t=?$ Answer: *6 с; 18 м/с.*

Task. At what angle is the tangent axis OX inclined to $y=x^3-x^2-7x+6$ at the point $(2;-4)$?

Answer: 45°.

Task. Write the equation of the tangent line to the graph $y=\frac{x^2-5}{x^2-4}$ at the point $x_0=1$.

Answer: $y=\frac{2}{9}x+1\frac{1}{9}$.

Task. Determine the dimensions of an outdoor pool with a square bottom volume 32 $м^3$ so that less material goes to the cladding of its walls and bottom.

Answer: *4 m, 2 m.*

Task. At what value of parameter a is the function $y=x^3-2{,}4x^2+ax-8{,}4$ has no extrema at critical points?

Answer: $a\neq 1{,}92$.