Section 14
Algebraic inequalities

Inequalities of degree higher than two are solved by the above method of intervals. By the same method, it is advisable to solve irregularities of the form:

$$\frac{\left(\sqrt{2^x}-8^{\frac{2}{3}}\right)\cdot(x-6)^2\cdot(x+3)}{\log_3(x+2)}\geq 0.$$

Solution:

We denote $f(x)=\dfrac{\left(\sqrt{2^x}-8^{\frac{2}{3}}\right)\cdot(x-6)^2\cdot(x+3)}{\log_3(x+2)}$.

We find the domain of the function $f(x)$:

$$\begin{cases} x+2>0, \\ \log_3(x+2)\neq 0. \end{cases} \quad \begin{cases} x>-2, \\ \log_3(x+2)\neq \log_3 1. \end{cases} \quad \begin{cases} x>-2, \\ x+2\neq 1. \end{cases} \quad \begin{cases} x>-2, \\ x\neq 1-2. \end{cases} \quad \begin{cases} x>-2, \\ x\neq -1. \end{cases}$$

$D(f)=(-2;-1)\cup(-1;+\infty)$.

Let us find zeros of the function $f(x)$: $\dfrac{\left(\sqrt{2^x}-8^{\frac{2}{3}}\right)\cdot(x-6)^2\cdot(x+3)}{\log_3(x+2)}=0.$

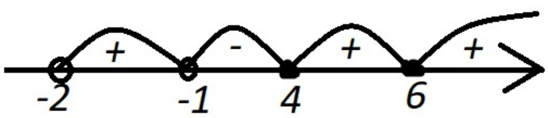
This equation is equivalent to such a system:

$$\begin{cases} \left[\begin{array}{l} \sqrt{2^x}-8^{\frac{2}{3}}=0, \\ (x-6)^2=0, \\ x+3=0, \end{array}\right. \\ x\in(-2;-1)\cup(-1;+\infty) \end{cases} \quad \begin{cases} \left[\begin{array}{l} 2^{\frac{x}{2}}-\left(2^3\right)^{\frac{2}{3}}=0, \\ x-6=0, \\ x+3=0, \end{array}\right. \\ x\in(-2;-1)\cup(-1;+\infty) \end{cases} \quad \begin{cases} \left[\begin{array}{l} 2^{\frac{x}{2}}=2^2, \\ x=6, \\ x=-3, \end{array}\right. \\ x\in(-2;-1)\cup(-1;+\infty) \end{cases}$$

$$\begin{cases} \left[\begin{array}{l} \frac{x}{2}=2, \\ x=6, \\ x=-3, \end{array}\right. \\ x\in(-2;-1)\cup(-1;+\infty) \end{cases} \quad \begin{cases} \left[\begin{array}{l} x=4, \\ x=6, \\ x=-3, \end{array}\right. \\ x\in(-2;-1)\cup(-1;+\infty) \end{cases} \quad \begin{array}{l} 4\in D(f), \\ 6\in D(f), \\ -3\notin D(f) \end{array}$$

Function zeros: $x=4$ and $x=6$.

On the number line, we denote the domain of definition of the function and in it we mark the zeros of the function:



On the number line, we denote the domain of definition of the function and in it we mark the zeros of the function: $f(x)$ in it:

$f(-1{,}5)=\dfrac{"-"\cdot"+"\cdot"+"}{"-"}>0;\ f(0)=\dfrac{"-"\cdot"+"\cdot"+"}{"+"}<0;\ f(5)=\dfrac{"+"\cdot"+"\cdot"+"}{"+"}>0;\ f(8)=\dfrac{"+"\cdot"+"\cdot"+"}{"+"}>0;$

In response to record those intervals on which the function $f(x)\geq 0$:

Answer: $(-2;-1)\cup[4;+\infty)$.

Fractional-rational inequalities are also solved by the method of intervals, but among them there are those that can be solved by a somewhat rational method, the so-called method of "snake". Its essence lies in the fact that they establish the range of permissible values, plot it on the number line. The left side of the inequality is denoted as a function of x, for example $f(x)$. Look for the roots of a function and mark them in the scope. In this case, the numbers that turn the denominator into zero are denoted by miniature circles, and the zeros of the function in the case of strict inequality - also circles, in the case of a non-strict inequality - by circles. If the exponent at the factor $(x-a_i)-$ is a paired number, then the number a is called zeros of paired multiplicity. It is advisable to underline the zeros of paired multiplicity in the figures.

Starting from the upper right edge, we have a line (the so-called snake) that passes through all the points indicated in the definition area. When passing through points of paired multiplicity, the "snake" does not pass from one number line, but goes to the next point. Above the interval, where the "snake" is located above the number line, write the sign "+", and where below "–".

Let us illustrate this with examples.

Solve inequality:

$$\frac{1}{x+1}-\frac{1}{x-1}\geq\frac{2}{3x+1}.$$

Solution:

$$\frac{1}{x+1}-\frac{1}{x-1}-\frac{2}{3x+1}\geq 0;\quad \frac{(3x+1)\cdot(x-1)-(x+1)\cdot(3x+1)\cdot 2\cdot(x+1)\cdot(x-1)}{(x+1)\cdot(x-1)\cdot(3x+1)}\geq 0;$$

$$\frac{3x^2-3x+x-1-3x^2-x-3x-1-2x^2+2}{(x+1)(x-1)(3x+1)}\geq 0;\quad \frac{-2x^2-6x}{(x+1)(x-1)(3x+1)}\geq 0;$$

$$\frac{-2x(x+3)}{(x+1)(x-1)\left(x+\frac{1}{3}\right)}\geq 0\,\Big|\times\left(-\frac{3}{2}\right);\quad \frac{x(x+3)}{\left(x+\frac{1}{3}\right)\cdot(x+1)\cdot(x-1)}\leq 0\ \textit{(A)}$$

We denote $f(x)=\dfrac{x(x+3)}{\left(x+\frac{1}{3}\right)(x+1)(x-1)}$.

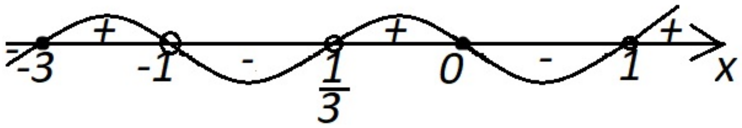
We are looking for the range of valid values: $\left(x+\frac{1}{3}\right)(x+1)(x-1)\neq 0.$

$$\left[\begin{array}{l} x+\dfrac{1}{3}\neq 0, \\ x+1\neq 0, \\ x-1\neq 0. \end{array}\right. \quad \left[\begin{array}{l} x\neq \dfrac{1}{3}, \\ x\neq -1, \\ x\neq 1. \end{array}\right.$$

The domain of the function $D(f)=(-\infty;-1)\cup\left(-1;-\dfrac{1}{3}\right)\cup\left(-\dfrac{1}{3};1\right)\cup(1;+\infty)$.

Looking for function zeros:

$$x\cdot(x+3)=0\rightarrow\left[\begin{array}{l} x=0, \\ x+3=0. \end{array}\right. \quad \left[\begin{array}{l} x=0, \\ x=-3. \end{array}\right.$$



Since there are no zeros of paired multiplicity of the function, we cross out the "snake" sequentially, starting from the upper right edge, through all points indicated in the definition area.
Taking into account the sign of inequality *(A)*, in response we write down the numerical intervals above which there is a sign «–».

Answer: $(-\infty;-3]\cup\left(-1;-\dfrac{1}{3}\right)\cup(0;1)$.

The "snake" rule is more convenient from the interval method in that there is no need to select one point from each interval and calculate the values of the function at these points.

$$\frac{x}{x+1}+\frac{x-2}{x^3+1}>\frac{2}{x^2-x+1}.$$

Solution:

$$\frac{x}{x+1}+\frac{x-2}{x^3+1}\frac{2}{x^2-x+1}>0;\rightarrow\frac{x}{x+1}+\frac{x-2}{(x+1)(x^2-x+1)}-\frac{2}{x^2-x+1}>0;$$

$$\rightarrow\frac{x(x^2-x+1)+x-2-2(x+1)}{(x+1)(x^2-x+1)}>0;\rightarrow\frac{x^3-x^2+x+x-2-2x-2}{(x+1)(x^2-x+1)}>0;$$

$$\rightarrow\frac{x^3-x^2-4}{(x+1)(x^2-x+1)}>0.$$

Spread the numerator x^3-x^2-4 by factors. It is easy to verify that the number 2 is the root of this equation, that is $x_1=2$.

$$
\begin{array}{l|l}
x^3-x^2-4 & x-2\\
-\underline{x^3-2x^2} & \overline{x^2+x+2}\\
\quad x^2-4 & \\
\quad -\underline{x^2-2x} & \\
\qquad 2x-4 & \\
\qquad -\underline{2x-4} & \\
\qquad\quad 0 &
\end{array}
$$

$x^2+x+2=0,\ D=1-8=-7<0,\ x\in\varnothing.$

This trinomial acquires only positive values.

Similarly, one can prove that:

$$x^2-x+1>0.$$

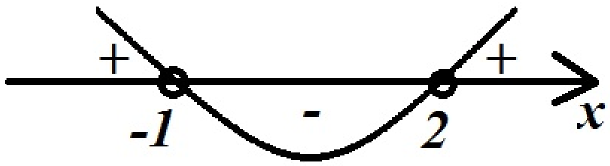
$$\frac{(x-2)\cdot(x^2+x+2)}{(x+1)\cdot(x^2-x+1)}>0 \ \Big|\times\frac{x^2-x+1}{x^2+x+2}$$

An equation is formed that is equivalent to the given:

$$\frac{x-2}{x+1}>0.$$

Range of valid values: $x+1\neq 0,\ x\neq -1.$

$D(f)=(-\infty;-1)\cup(-1;+\infty).$



Zero of function $x-2=0,\ \ x=2.$

There are no roots of paired multiplicity of the function. Starting from the top right edge, cross out the "snake".

Answer: $(-\infty;-1)\cup(2;+\infty).$

$$\frac{(-x^2-1)\cdot(x-1)^3\cdot(x^2-4)^2\cdot(x^2-9)^5}{(1+3x)\cdot(x^2-x-6)}\leq 0.$$

Solution:

$$\frac{-(x^2+1)\cdot(x-1)^3\cdot(x-2)^2\cdot(x^2+2)^2\cdot(x-3)^5\cdot(x+3)^5}{3\cdot\left(\frac{1}{3}+x\right)\cdot(x^2-x-6)}\leq 0\Big|\times(-3);$$

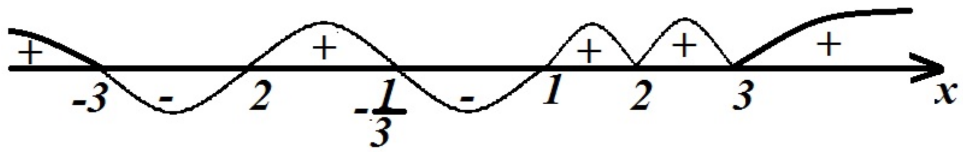
$$\frac{(x^2+1)\cdot(x-1)^3\cdot(x-2)^2\cdot(x+2)^2\cdot(x-3)^5\cdot(x+3)^5}{\left(x+\frac{1}{3}\right)\cdot(x-3)(x+2)}\geq 0.$$

Because the $\begin{matrix}x-3\neq 0,\\ x+2\neq 0\end{matrix}$ then you can shorten:

$$\frac{(x^2+1)\cdot(x-1)^3\cdot(x-2)^2\cdot(x+2)\cdot(x-3)^4\cdot(x+3)^5}{x+\frac{1}{3}}\geq 0.$$

Domain of the function $(-\infty;-2)\cup\left(-2;-\frac{1}{3}\right)\cup\left(-\frac{1}{3};3\right)\cup(3;+\infty),\ x^2+1>0$ for all values of x. We divide both sides of the inequality by (x^2+1):

$$\frac{(x-1)^3\cdot(x-2)^2\cdot(x+2)\cdot(x-3)\ \cdot(x+3)^5}{x+\frac{1}{3}}\geq 0$$



Considering the fact that 2 and 3 are parity zeros of the function, cross out the "snake".

Answer: $(-\infty;-3]\cup\left(-2;-\frac{1}{3}\right)\cup[1;3)\cup(3;+\infty)$.

$$\frac{(x^2-x-2)\cdot(x^2+5x+6)^3\cdot(2x+5)^4}{(x^2-2x-3)^2\cdot(x^2-x-20)}\leq 0.$$

Solution:

Using Vieta's theorem, factor the square trinomials:

$x^2-x-2=(x-2)\cdot(x+1);\ \ x^2+5x+6=(x+2)\cdot(x+3);\ \ x^2-2x-3=(x-3)(x+1);$

$x^2-x-20=(x+4)(x-5).$

Then this inequality will have the form:

$$\frac{(x-2)\cdot(x+1)\cdot((x+2)\cdot(x+3))^3\cdot(2\cdot(x+2{,}5))^4}{((x-3)(x+1))^2\cdot(x+4)(x-5)}\leq 0;$$

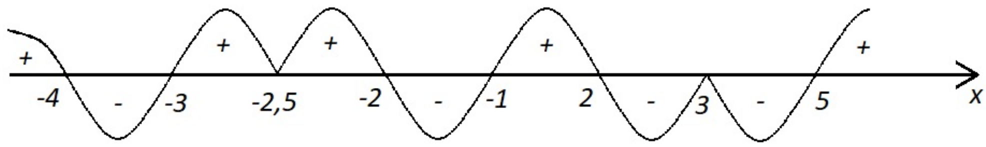
$$\frac{(x-2)\cdot(x+1)\cdot(x+2)^3\cdot(x+3)^3\cdot 16\cdot(x+2{,}5)^4}{(x-3)^2\cdot(x+1)^2\cdot(x+4)\cdot(x-5)}\leq 0;$$

$D(f)=(-\infty;-4)\cup(-4;-1)\cup(-1;3)\cup(3;5).$

Because $x+1\neq 0,$ then we will reduce by this expression:

$$\frac{(x-2)\cdot(x+2)^3\cdot(x+3)^3\cdot 16\cdot(x+2{,}5)^4}{(x-3)^2\cdot(x+1)\cdot(x+4)\cdot(x-5)}\leq 0;$$

Zeros of function $x=2;\ x=-2;\ x=-3;\ x=-2{,}5.$



Considering that the numbers –2.5 and 3 are zeros of paired multiplicity, cross out the "snake".

Answer: $(-4;-3)\cup\{-2{,}5\}\cup[-2;-1)\cup[2;3)\cup(3;5).$

$(x^2-4)\cdot(x^2-4x+4)\cdot(x^2-6x+8)\cdot(x^2+4x+4)\cdot(x^2-4x+5)\leq 0.$

Solution:

Factor the expressions in parentheses:

$x^2-4=(x-2)\cdot(x+2);\ \ x^2-4x+4=(x-2)^2;\ \ x^2-6x+8=0,\ \ x_1=2;\ \ x_2=4.$

$x^2-6x+8=(x-2)\cdot(x-4);\ \ x^2+4x+4=(x+2)^2;\ \ x^2-4x+5=0;\ \ D=16-25=-9<0$

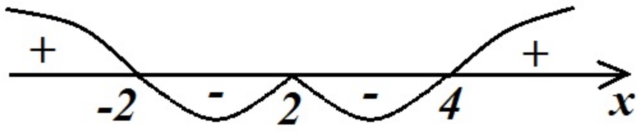
$x^2-4x+5>0$ for all values of x. Then

$(x-2)\cdot(x+2)\cdot(x-2)^2\cdot(x-2)\cdot(x-4)\cdot(x+2)^2\cdot(x^2-4x+5)\leq 0;$

$(x-2)^4 \cdot (x+2)^3 \cdot (x-4) \cdot (x^2-4x+5) \le 0 \,\big|: (x^2-4x+5);$

$(x-2)^4 \cdot (x+2)^3 \cdot (x-4) \le 0; \;\; D(f) = (-\infty;+\infty).$

Zeros of function: $x=2-$ paired multiplicity; $x=-2;$ $x=4.$



$x \in [-2;2] \cup [2;4].$

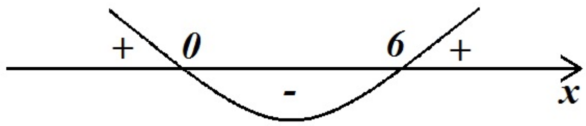
Answer: $[-2;4].$

$(x^2-x-6)^2 - 6 \cdot (x^2-x-5) + 6 \ge 0.$

Solution:

Let's introduce a new variable $t = x^2 - x - 6 \,|+1; \;\; t+1 = x^2 - x - 5.$

Then $t^2 - 6 \cdot (t+1) + 6 \ge 0,$ $t^2 - 6t - 6 + 6 \ge 0,$ $t^2 - 6t \ge 0,$ $t(t-6) \ge 0,$ $t=0,$ $t=6-$ zeros of function.



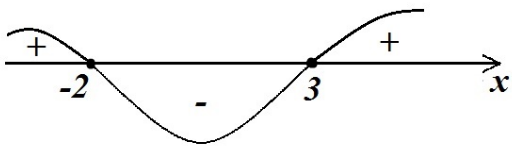
$t \in (-\infty;0] \cup [6;+\infty).$

Using the replacement, we obtain the set of inequalities:

$\left[\begin{array}{l} x^2-x-6 \le 0, \\ x^2-x-6 \ge 0. \end{array}\right. \;\; \left[\begin{array}{l} x^2-x-6 \le 0, \\ x^2-x-12 \ge 0. \end{array}\right. \;\; \left[\begin{array}{l} (x-3)(x+2) \le 0, \\ (x-4)(x+3) \ge 0. \end{array}\right.$

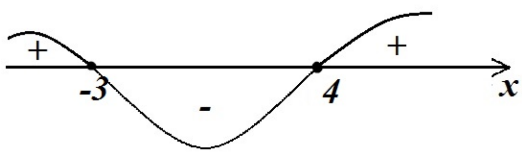
We solve each unevenness of the set using the "snake" method:

$(x-3)(x+2) \le 0$



$x \in [-2;3]$

$(x-4)(x+3) \ge 0$



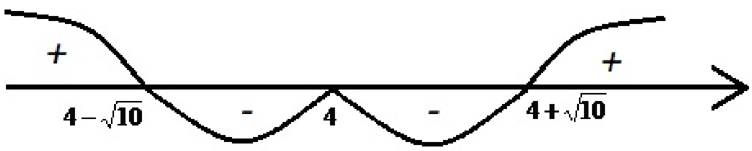
$x \in (-\infty;-3] \cup [4;+\infty)$

Answer: $(-\infty;-3] \cup [-2;3] \cup [4;+\infty).$

$(x-1) \cdot (x-3) \cdot (x-5) \cdot (x-7) > 9.$

Solution:

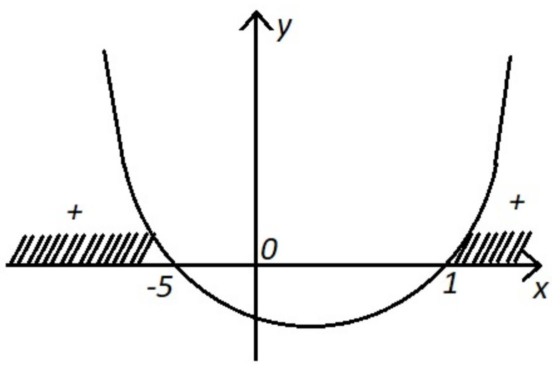
Let's multiply the two extreme factors and two middle ones:
$(x-1)\cdot(x-7)\cdot(x-3)\cdot(x-5)=\left(x^2-7x-x+7\right)\cdot\left(x^2-5x-3x+15\right)=$
$=\left(x^2-8x+7\right)\cdot\left(x^2-8x+15\right)$. Тоді $\left(x^2-8x+7\right)\cdot\left(x^2-8x+15\right)-9>0$. (A)
Let's introduce a new variable: $x^2-8x+7=t+8$, $x^2-8x+15=t+8$.
Inequality (A) takes the form:
$t\cdot(t+8)-9>0$, $t^2+8t-9>0$, $t_1=-9$, $t_2=1$.
An unexpected move: $t^2+8t-9=(t+9)\cdot(t-1)$.
$(t+9)\cdot(t-1)=\left(x^2-8x+7+9\right)\cdot\left(x^2-8x+7-1\right)=\left(x^2-8x+16\right)\cdot\left(x^2-8x+6\right)=$
$=(x-4)^2\cdot\left(x-4+\sqrt{10}\right)\cdot\left(x+4+\sqrt{10}\right)$;
$D=64-24=40=4\cdot 10$.
$x_1=\dfrac{8-2\sqrt{10}}{2}=4-\sqrt{10}$; $x_2=4+\sqrt{10}$.
Then $(x-4)^2\cdot\left(x-\left(4-\sqrt{10}\right)\right)\cdot\left(x+\left(4+\sqrt{10}\right)\right)>0$.
We denote $f(x)=(x-4)^2\cdot\left(x-\left(4-\sqrt{10}\right)\right)\cdot\left(x+\left(4+\sqrt{10}\right)\right)$.
The zeros of this function are numbers:
$x=4$, $x=4-\sqrt{10}$, $x=4+\sqrt{10}$.
Solving inequality using the "snake" method:



Answer: $\left(-\infty;4-\sqrt{10}\right)\cup\left(4+\sqrt{10};+\infty\right)$.

<u>Иррациональные неравенства</u>

It is convenient to solve them by the method of intervals:
$$\frac{\sqrt{x^2+4x-5}}{x+1}<1.$$
Solution:
$$\frac{\sqrt{x^2+4x-5}}{x+1}-1<0;\quad \frac{\sqrt{x^2+4x-5}-x-1}{x+1}<0;$$
We denote $f(x)=\dfrac{\sqrt{x^2+4x-5}-x-1}{x+1}$.
We find the domain of the function $f(x)$:
$$\begin{cases} x^2+4x-5\ge 0,\\ x+1\ne 0.\end{cases}$$
By Vieta's theorem $x_1=-5$, $x_2=1$.
Then $\begin{cases}(x-1)\cdot(x+5)\ge 0,\\ x\ne -1.\end{cases}$



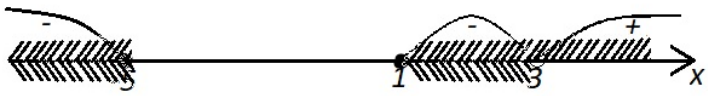
$D(f)=(-\infty;-5]\cup[1;+\infty)$.

Find the zeros of the function $f(x)$: $f(x)=0$,

$$\frac{\sqrt{x^2+4x-5}-x-1}{x+1}=0,\quad \begin{cases}\sqrt{x^2+4x-5}-x-1=0,\\ x+1\neq 0.\end{cases}\quad \begin{cases}\sqrt{x^2+4x-5}=x+1,\\ x\neq -1.\end{cases}$$

Let's square both sides of the equation:

$$\begin{cases}x^2+4x-5=(x+1)^2,\\ x+1\geq 0,\\ x\neq -1.\end{cases}\quad \begin{cases}x^2+4x-5=x^2+2x+1,\\ x>-1.\end{cases}\quad \begin{cases}2x-6=0,\\ x>-1.\end{cases}\quad \begin{cases}x=3,\\ x>-1.\end{cases}$$



Shade the domain of the function $f(x)$.

Let us denote in it $x=3-$ zero of function.

Let us mark the intervals formed and determine the sign of the function $f(x)$ on each of them:

$$f(-8)=\frac{\sqrt{64-32-5}+8-1}{-8+1}=\frac{\sqrt{27}+7}{-7}=\frac{"+"}{"-"}<0,$$

$$f(2)=\frac{\sqrt{2^2+8-5}-2-1}{2+1}=\frac{\sqrt{7}-3}{3}=\frac{"-"}{"+"}<0,$$

$$f(5)=\frac{\sqrt{25+20-5}-5-1}{5+1}=\frac{\sqrt{40}-6}{6}=\frac{"+"}{"+"}>0.$$

Let's shade the intervals at which the function takes negative values. In response, write down the intervals at which the "herringbone" was formed.

Answer: $(-\infty;-5]\cup[1;3)$.

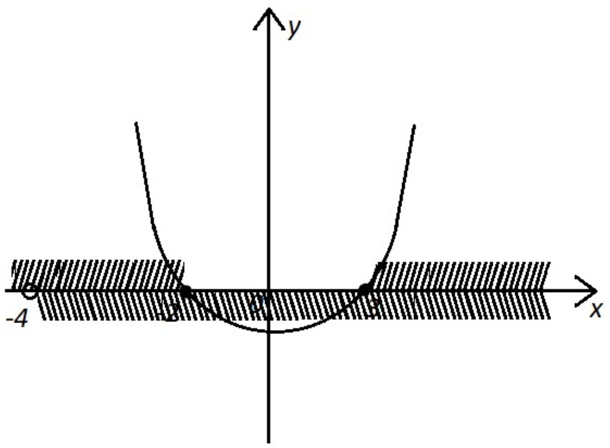
There are a large number of irrational inequalities that it is advisable to solve by going to equivalent systems or sets of systems of simple inequalities.

$$\sqrt[4]{x^2-x-6}<\sqrt[4]{x^2+3x+10}.$$

Solution:

Raise both sides of the inequality to the 4th power:

$$\begin{cases}x^2-x-6<x^2+3x+10,\\ x^2-x-6\geq 0.\end{cases}\quad \begin{cases}-4x<16,\,|:(-4)\\ (x-3)\cdot(x+2)\geq 0.\end{cases}\quad \begin{cases}x<-4,\\ (x-3)\cdot(x+2)\geq 0.\end{cases}$$



Answer: $(-4;-2]\cup[3;+\infty)$.

$\sqrt{2x-8}>\sqrt{7-x}$.

Solution:

$$\begin{cases}2x-8>7-x,\\7-x\geq 0.\end{cases}\quad \begin{cases}3x>15,\\x\leq 7.\end{cases}\quad \begin{cases}x>5,\\x\leq 7.\end{cases}$$

Answer: $(5;7]$.

$\sqrt{4x-x^2}<4-x$.

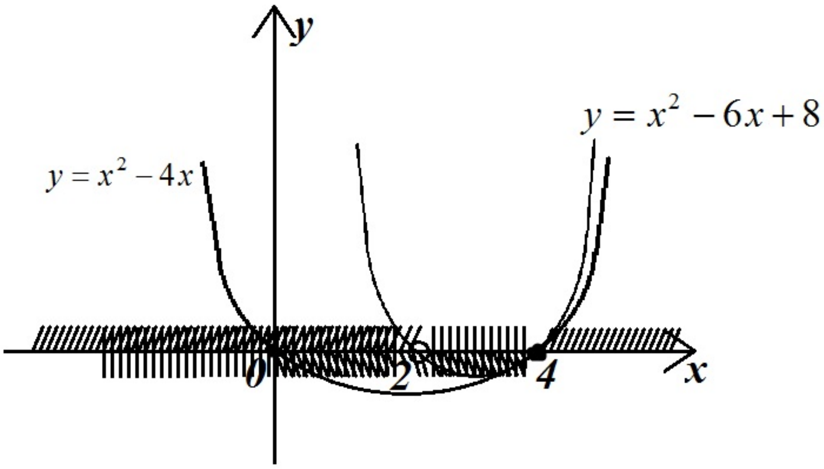
Solution:

$$\begin{cases}4x-x^2<(4-x)^2,\\4x-x^2\geq 0,\\4-x\geq 0.\end{cases}\quad \begin{cases}4x-x^2<16-8x+x^2,\\x(4-x)\geq 0|\cdot(-1),\\x\leq 4.\end{cases}\quad \begin{cases}-2x+12x-16<0|:(-2),\\x\cdot(x-4)\leq 0,\\x\leq 4.\end{cases}\quad \begin{cases}x^2-6x+8>0,\\(x-4)x\leq 0,\\x\leq 4.\end{cases}$$

By Vieta's theorem, we have: $x_1=2,\quad x_2=4,$ By Vieta's theorem, we have:

$$\begin{cases}(x-2)\cdot(x-4)>0,\\x\cdot(x-4)\leq 0,\\x\leq 4.\end{cases}$$

In our opinion, there pertinent graphic illustration of the system of inequalities solutions.



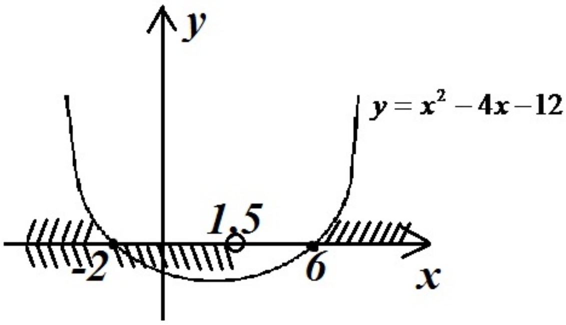
Answer: $[0;2)$.

$\sqrt{x^2-4x-12} > 2x-3.$

Solution:

$$\left[\begin{array}{l}\begin{cases}2x-3\ge 0,\\ x^2-4x-12>(2x-3)^2.\end{cases}\\ \begin{cases}2x-3<0,\\ x^2-4x-12\ge 0.\end{cases}\end{array}\right. \quad \left[\begin{array}{l}\begin{cases}x\ge 1{,}5,\\ x^2-4x-12>4x^2-12x+9.\end{cases}\\ \begin{cases}x<1{,}5,\\ (x-6)(x+2)\ge 0.\end{cases}\end{array}\right. \quad \left[\begin{array}{l}\begin{cases}x\ge 1{,}5,\\ -3x^2+8x-21>0 \,|\cdot(-1)\end{cases}\\ \begin{cases}x<1{,}5,\\ (x-6)(x+2)\ge 0.\end{cases}\end{array}\right.$$

$$\begin{cases}x\ge 1{,}5,\\ 3x^2-8x+21<0.\end{cases} \quad \begin{cases}D=64-252<0,\\ x\ge 1{,}5.\end{cases} \quad \begin{cases}x<1{,}5,\\ (x-6)(x+2)\ge 0.\end{cases}$$

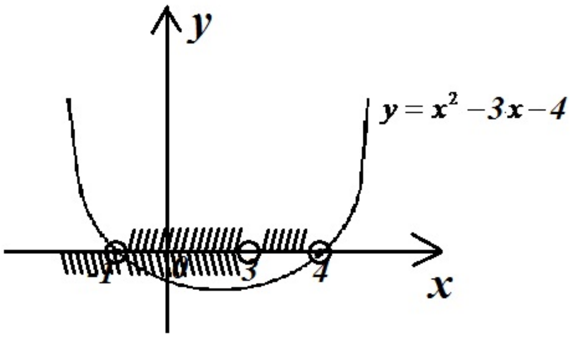


Answer: $(-\infty;-2]$.

$(3-x)\cdot\sqrt{4+3x-x^2} > 0.$

Solution:

$$\begin{cases}3-x>0,\\ 4+3x-x^2>0 \,|\cdot(-1)\end{cases} \quad \begin{cases}x<3,\\ x^2-3x-4<0\end{cases} \quad \begin{cases}x<3,\\ (x+1)(x-4)<0.\end{cases}$$



Answer: $(-1;3)$.

$$\frac{x^2-13x+40}{\sqrt[6]{19x-x^2-78}} \le 0.$$

Solution:

Because the $\sqrt[6]{19x-x^2-78} \ge 0$, then

$$\begin{cases}x^2-13x+40\le 0,\\ 19x-x^2-78>0.\end{cases}$$

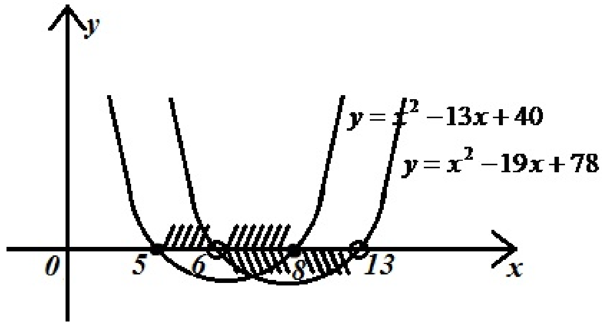
$x^2-13x+40=0$ by Vieta's theorem $x_1=5,\ x_2=8,$ then $x^2-13x+40=(x-5)(x-8)$

$-x^2+19x-78=0,\ \ x^2-19x+78=0,\ \ x_1=6,\ \ x_2=13.$

$x^2-19x+78=(x-6)(x-13)$

The system of inequalities will look like:

$$\begin{cases}(x-5)(x-8)\le 0,\\ -\left(x^2-19x+78\right)>0 \,|\cdot(-1)\end{cases} \begin{cases}(x-5)(x-8)\le 0,\\ x^2-19x+78>0\end{cases} \begin{cases}(x-5)(x-8)\le 0,\\ (x-6)(x-13)<0.\end{cases}$$



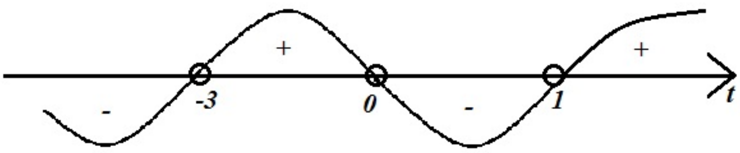
Answer: $(6;8]$.

$$\frac{3}{\sqrt{x-2}}-\sqrt{2-x}<2.$$

Solution:

We denote $\sqrt{2-x}=t,\ t>0.$ Then $\frac{3}{t}-t-2<0,\ \frac{-t^2-2t+3}{t}<0,|\cdot(-1)$

$\frac{t^2+2t-3}{t}>0,\ \frac{(t-1)(t+3)}{t}>0.$



$t\in(-3;0)\cup(1;+\infty).$ Returning to the replacement, we have:

$-3<\sqrt{2-x}<0,\ \sqrt{2-x}\ge 0, x\in\varnothing.$

$\sqrt{2-x}>1.$ Raise to the square:

$2-x>1, -x>-1, x<1.\ \ x\in(-\infty;1).$

Answer: $(-\infty;1).$