Section 7
Exponential Equations

Today it is difficult to overestimate the role and place of exponential equations in the course of mathematics, not only as a theoretical, but also applied science. Solution of exponential equations for those who solve them makes it possible to better repeat many areas of mathematics. In addition, the properties of the exponential function are specifically and consciously fixed, which facilitates their application to solving problems of an applied nature.

Exponential to be called a kind $a^x = в$, where a>0 and $a \neq 1$.

Before solving the exponential equations, one should understand well the following theorem: if a > 0 and $a \neq 1$. Then with equality $a^m = a^n$ equality follows $m = n$. Solution the simplest exponential equations are expediently reduced to one basis.

For example, solve the equation $0{,}5^x = \frac{1}{128}$.

Solution:

The left and right sides of the equation reduces to the base $\frac{1}{2}$: $0{,}5 = \frac{1}{2}$; $\frac{1}{128} = \left(\frac{1}{2}\right)^7$,

then the equation will have the form: $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^7 \Rightarrow x = 7$.

In the same manner solved equation of the form $\left(\frac{9}{4}\right)^x \cdot \left(\frac{2}{3}\right)^{x-2} = \left(\frac{8}{27}\right)^{3-5x}$.

Solution:

Let's perform identical transformations of expressions:

$\left(\frac{9}{4}\right)^x = \left(\frac{3^2}{2^2}\right)^x = \left(\frac{3}{2}\right)^{2x} = \left(\frac{2}{3}\right)^{-2x}$; $\left(\frac{8}{27}\right)^{3-5x} = \left(\frac{2^3}{3^3}\right)^{3-5x} = \left(\frac{2}{3}\right)^{9-15x}$, then

$\left(\frac{2}{3}\right)^{x-2} \cdot \left(\frac{2}{3}\right)^{-2x} = \left(\frac{2}{3}\right)^{9-15x}$; $\left(\frac{2}{3}\right)^{x-2-2x} = \left(\frac{2}{3}\right)^{9-15x}$;

Due to the monotony of the exponential function have:

$-2 - x = 9 - 15x$; $14x = 11$; $x = \frac{11}{14}$.

Answer: $x = \frac{11}{14}$.

Let us show solutions to more complex exponential equations:

$$\left(2+\sqrt{3}\right)^{x^2-2x+1} + \left(2-\sqrt{3}\right)^{x^2-2x-1} = \frac{4}{2-\sqrt{3}}.$$

Solution:

We multiply both sides of the equation by this expression: $\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right)^{x^2-2x} > 0$.

Then:

$$\left(2+\sqrt{3}\right)^{x^2-2x+1}\cdot\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right)^{x^2-2x}+\left(2-\sqrt{3}\right)^{x^2-2x-1}\cdot\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right)^{x^2-2x}=\frac{4}{2-\sqrt{3}}\cdot\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right)^{x^2-2x}.$$

$$\left(2+\sqrt{3}\right)^{x^2-2x}\cdot\left(2+\sqrt{3}\right)\cdot\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right)^{x^2-2x}+\frac{\left(2-\sqrt{3}\right)^{x^2-2x}}{2-\sqrt{3}}\cdot\left(2-\sqrt{3}\right)\cdot\left(2+\sqrt{3}\right)^{x^2-2x}=$$

$$=\frac{4\cdot\left(2-\sqrt{3}\right)}{2-\sqrt{3}}\cdot\left(2+\sqrt{3}\right)^{x^2-2x};$$

$$\left(\left(2+\sqrt{3}\right)^{x^2-2x}\right)^2\cdot(4-3)+(4-3)^{x^2-2x}=4\cdot\left(2+\sqrt{3}\right)^{x^2-2x};\quad \left(\left(2+\sqrt{3}\right)^{x^2-2x}\right)^2\cdot 1-4\cdot\left(2+\sqrt{3}\right)^{x^2-2x}+1=0.$$

Replacement: $\left(2+\sqrt{3}\right)^{x^2-2x}=t>0,$ then $t^2-4t+1=0.\quad D=16-4=12=4\cdot 3.$

$$t_1=\frac{4-\sqrt{4\cdot 3}}{2}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}=\left(2+\sqrt{3}\right)^{-1};\quad t_2=\frac{4+2\sqrt{3}}{2}=2+\sqrt{3}.$$

A set of equations is formed: $\left[\begin{array}{l}\left(2+\sqrt{3}\right)^{x^2-2x}=\left(2+\sqrt{3}\right)^{-1},\\ \left(2+\sqrt{3}\right)^{x^2-2x}=\left(2+\sqrt{3}\right).\end{array}\right.$

Since the exponential function is monotonic, then we solve this set of equations:

$$\left[\begin{array}{l}x^2-2x=-1,\\ x^2-2x=1.\end{array}\right.\quad \left[\begin{array}{l}x^2-2x+1=0,\\ x^2-2x-1=0.\end{array}\right.\quad \left[\begin{array}{l}(x-1)^2=0,\\ D=4+4=8=4\cdot 2.\end{array}\right.$$

$$x_1=1;\quad x_2=\frac{2-2\sqrt{2}}{2}=1-\sqrt{2};\quad x_3=1+\sqrt{2}.$$

Answer: $1-\sqrt{2};\ 1;\ 1+\sqrt{2}.$

Equation of this type $3^{x+1}+3^{x-1}+3^{x-2}=5^x+5^{x-1}+5^{x-2}$, are solved by taking the common factor out of parentheses.

Solution:

$5^{x-1}=5^x:5=\frac{1}{5}\cdot 5^x;\quad 5^{x-2}=5^x\cdot 5^{-2}=\frac{1}{25}\cdot 5^{-x};$ Then the equation can be rewritten as follows:

$$5^x\cdot 3+3^x\cdot\frac{1}{3}+3^x\cdot\frac{1}{3}+3^x\cdot\frac{1}{25}=\left(5^x+5^x\cdot\frac{1}{5}+5^x\cdot\frac{1}{25}\right);\quad 3^x\cdot\left(3+\frac{1}{3}+\frac{1}{9}\right)=5^x\cdot\left(5+\frac{1}{5}+\frac{1}{25}\right)=3^x\cdot\frac{31}{9}=5^x\cdot\frac{31}{25}\Big|:\left(5^x\cdot\frac{31}{25}\right);$$

$$\frac{3^x\cdot\frac{31}{9}}{5^x\,\frac{31}{25}}=1;\quad \left(\frac{25}{9}\right)^x\cdot\frac{25}{9}=1;\quad \left(\frac{3}{5}\right)^x=\frac{9}{25};\quad \left(\frac{3}{5}\right)^x=\left(\frac{3}{5}\right)^2;\quad x=2.$$

Answer: 2.

Sometimes a function that contains a variable in an exponent is called an ***exponent***, and the law that it characterizes is called an ***exponential***. So, according to the exponential law, bacteria multiply and the radioactive decay of uranium nuclei occurs. The exponential law helps in studying the anode characteristics of vacuum tubes. Differential equations of harmonic oscillations are at the heart of electrical radio technology. The nature of exponential equations has led to great attention to them, and this contributes to the emergence of more and more new ways to solve them..

Solve the equation: $25^x-10^x=2^{2x+1}.$

Solution:

$25^x - 10^x = 4^x \cdot 2 \,\big|\, : (4^x);\quad \frac{25^x}{4^x} - \frac{10^x}{4^x} = \frac{4^x \cdot 2}{4^x};\quad \frac{(5^2)^x}{(2^2)^x} - \left(\frac{5}{2}\right)^x = 2;\quad \left(\frac{5}{2}\right)^{2x} - \left(\frac{5}{2}\right)^x = 2;$

We denote $\left(\frac{5}{2}\right)^x = y,$ then the equation will have the form: $y^2 - y - 2 = 0;$

$y_1 = 2,\ y_2 = (-1);$

$\left(\frac{5}{2}\right)^x > 0,$ and therefore the equation $\left(\frac{5}{2}\right)^x = (-1) -$ has no valid roots. $\left(\frac{5}{2}\right)^x = 2.$

Let us logarithm both parts with the base $\frac{5}{2}$; $\log_{\frac{5}{2}}\left(\frac{5}{2}\right)^x = \log_{\frac{5}{2}} 2;\quad x \cdot 1 = \log_{\frac{5}{2}} 2.$

Answer: $\log_{\frac{5}{2}} 2.$

Consider such a combined equation: $10^{\sin^2 x} + 10^{\cos^2 x} = 11.$

Solution:

After some identity transformations of this equation takes the form:

$10^{\cos^2 x} = 10^{1-\sin^2 x} = \frac{10}{10^{\sin^2 x}};\quad 10^{\sin^2 x} + \frac{10}{10^{\sin^2 x}} = 11;$

We introduce such a complex replacement:

$10^{\sin^2 x} = y > 0;\quad y + \frac{10}{y} - 11y = 0;\quad y^2 - 11y + 10 = 0;\quad y_1 = 1;\ y_2 = 10.$

Consider the set of equations:

$$\left[\begin{array}{l} 10^{\sin^2 x} = 1; \\ 10^{\sin^2 x} = 10. \end{array}\right. \quad \left[\begin{array}{l} 10^{\sin^2 x} = 10^0; \\ 10^{\sin^2 x} = 1. \end{array}\right. \quad \left[\begin{array}{l} \sin^2 x = 0; \\ \sin^2 x = 11. \end{array}\right. \quad \left[\begin{array}{l} x = \pi \cdot n,\ n \in Z; \\ \sin^2 x = 11; \\ \sin^2 x = -1. \end{array}\right. \quad \left[\begin{array}{l} x = \pi \cdot n; \\ x = \frac{\pi}{2} + 2\pi \cdot n; \\ x = -\frac{\pi}{2} + 2\pi \cdot n. \end{array}\right.$$

Answer: $\pi \cdot n;\ \frac{\pi}{2} + 2\pi \cdot n;\ -\frac{\pi}{2} + 2\pi \cdot n;\ n \in Z.$

The process of solving some logarithmic equations is greatly simplified if the original equation is replaced with the corresponding exponential equation.

$\frac{1}{2} \cdot \lg x + 3 \cdot \lg \cdot \sqrt{2+x} = \lg \cdot \sqrt{x \cdot (x+2)} + 2.$

Solution:

Find range of valid values: $\left[\begin{array}{l} x > 0; \\ \sqrt{2+x} > 0. \end{array}\right. \quad \left[\begin{array}{l} x > 0; \\ 2 + x > 0. \end{array}\right. \quad \left[\begin{array}{l} x > 0; \\ x > -2. \end{array}\right. \quad x > 0.$

$\frac{1}{2} \cdot \lg x + 3 \cdot \lg \cdot \sqrt{2+x} = \lg \cdot \sqrt{x \cdot (x+2)} + 2.$

$\frac{1}{2} \cdot \lg x + 3 \cdot \lg \cdot \sqrt{2+x} - \frac{1}{2} \lg \sqrt{x} - \lg \sqrt{x+2} = 2;$

$3 \cdot \lg \sqrt{2+x} - 2 \lg \sqrt{x+2} = 2;$

$\lg \sqrt{x+2} = 1;\quad \sqrt{x+2} = 10^1;\quad x + 2 = 100;\quad x = 100 - 2;\quad x = 98.$

Answer: 98.
The way to solve certain equations significantly reduced if they timely move from one to the other bases of logarithms.
For example: $\sqrt{1+\log_x \sqrt{27}}\cdot\log_3 x+1=0.$

Solution:

Proceed to the base of the logarithms 3: $\log_3 \sqrt{27}=\frac{\log_3 \sqrt{27}}{\log_3 x}=\frac{\log_3 3^{\frac{3}{2}}}{\log_3 x}=\frac{3}{2\cdot\log_3 x}.$

Then the equation takes the form: $\sqrt{1+\frac{3}{2\log_3 x}}\cdot\sqrt{(\log_3 x)^2}+1=0;$

$\sqrt{\log_3{}^2 x+\frac{3}{2}\log_3 x}+1=0;\quad \log_3{}^2 x+\frac{3}{2}\log_3 x=(-1)^2;\quad \log_3{}^2 x+\frac{3}{2}\log_3 x=1;$

$\log_3{}^2 x+\frac{3}{2}\log_3 x-1=0\,\Big|\cdot 2;\quad 2\log_3{}^2 x+3\log_3 x-2=0;\quad D=9+16=25;$

$\log_3 x=\frac{-3-5}{4}=-2;\quad x=3^{-2}=\frac{1}{9};\quad \log_3 x=\frac{-3+5}{2}=\frac{2}{4}=\frac{1}{2};\quad x=\sqrt{3}.$

A direct check shows that the number $\sqrt{3}$ does not satisfy the original equation, and $\frac{1}{9}$ satisfies him. Well then, Answer $\frac{1}{9}$.

Answer: $\frac{1}{9}$.

Solve the equation: $3^{2x+4}+45\cdot 6^x-9\cdot 2^{2x+2}=0.$

Solution:

$3^{2x+4}+45\cdot 6^x-9\cdot 2^{2x+2}=0\,\Big|:6^x;\quad \frac{3^{2x+4}}{3^x\cdot 2^x}+\frac{45\cdot 6^x}{6^x}-9\cdot\frac{2^{2x+2}}{3x\cdot 2x}=0;\quad \frac{3^{2x+4}}{2^x}+45-9\cdot\frac{2^{2x+2-x}}{3x}=0;$

$\frac{3^x\cdot 3^4}{2^x}+45-9\cdot\frac{2^{2x+2}}{3x}=0;\quad 81\cdot\left(\frac{3}{2}\right)^x-9\cdot 4\cdot\left(\frac{2}{3}\right)^x=0.$

Replacement $\left(\frac{3}{2}\right)^x=y$ leads to the equation:

$81y-36\cdot\frac{1}{y}+45=0\,\Big|\cdot y;\quad 81y^2+45y-36=0\,\Big|:9;\quad 9y^2+5y-4=0;\quad D=25+144=169,$

$y_1=\frac{-5-13}{18}=-1;\ y_2=\frac{-5+13}{18}=\frac{4}{9}.\ \left(\frac{3}{2}\right)^x=-1,\ x\in\varnothing$ in range of R.

$\left(\frac{3}{2}\right)^x=\frac{4}{9};\ \left(\frac{3}{2}\right)^x=\left(\frac{9}{4}\right)^{-1};\ \left(\frac{3}{2}\right)^x=\left(\frac{3}{2}\right)^{-2};\ x=-2.$ Answer: -2.

When solving exponential equations, the difference cube formula will be a good lever in this form: $(a-b)^3=(a-b)^3=a^3-b^3-3ab\cdot(a-b).$
Let us illustrate what has been said by solving such an equation:
$2^{3x}-2^{3-3x}=1+6\cdot\left(2^x-\frac{1}{2^{x-1}}\right).$

Solution:

$2^{3x} = \left(2^x\right)^3; \quad 2^{3-3x} = \left(1-2^{1-x}\right)^3; \quad \frac{1}{2^{x-1}} = 2^{1-x};$

Then the original equation will have the form:

$\left(2^x\right)^3 - \left(2^{1-x}\right)^3 = 1 + 6\cdot\left(2^x - 2^{1-x}\right); \quad \left(2^x\right)^3 - \left(2^{1-x}\right)^3 - 6\cdot\left(2^x - 2^{1-x}\right) = 1; \quad 6 = 3\cdot 2 = 3\cdot \underbrace{2^x \cdot 2^{1-x}}_{2^{x+1-x}=2^1=2.}$

$\left(2^x\right)^3 - \left(2^{1-x}\right)^3 - 3\cdot 2^x \cdot 2^{1-x}\cdot\left(2^x - 2^{1-x}\right) = 1;$ Which means, $\left(2^x - 2^{1-x}\right)^3 = 1; \quad 2^x - \frac{2}{2x} = \frac{(2x)^2 - 2}{2x};$

$2^x > 0.$

$\left(\frac{\left(2^x\right)^2 - 2}{2^x}\right)^3 = 1^3; \quad \frac{\left(2^x\right)^2 - 2}{2^x} 1; \quad \left(2^x\right)^2 - 2^x - 2 = 0;$ We denote $2^x = y,$ then the reduced quadratic equation will have the form: $y^2 - y - 2 = 0$ its roots $y_1 = -1,\ y_2 = 2.$

Returning to the replacement, we have: $2^x = -1,\ x \in \varnothing. \quad 2^x = 2 \Rightarrow x = 1.$

Answer: 1.

Exponential equations like $4^x + 6^x = 9^x$, after simple transformations are reduced to quadratic equations.

Solution:

Considering that $4^x = 2^{2x}; \quad 6^x = 2^x \cdot 3^x; \quad 9^x = 3^{2x}.$ We have: $2^{2x} + 2^x \cdot 3^x - 3^{2x} = 0.$

Dividing both sides of this equation by 3^{2x} we come to a quadratic equation for $\left(\frac{2}{3}\right)^x$: $\frac{2^{2x}}{3^{2x}} + \frac{2^x\cdot 3^x}{3^{2x}} - \frac{3^{2x}}{3^{2x}}; \quad \left(\frac{2}{3}\right)^{2x} + \left(\frac{2}{3}\right)^x - 1 = 0; \ D = 1 + 4 = 5; \quad \left(\frac{2}{3}\right)^x = \frac{-1-\sqrt{5}}{2\cdot 1}$ - this is less than zero, so $x \in \varnothing$ in R. $\left(\frac{2}{3}\right)^x = \frac{-1+\sqrt{5}}{2} < 0,\ x = \log_{\frac{2}{3}} \frac{\sqrt{5}-1}{2}.$

Answer: $x = \log_{\frac{2}{3}} \frac{\sqrt{5}-1}{2}.$

Interestingly solved equations of the type: $\left(\sqrt{2-\sqrt{3}}\right)^x + \left(\sqrt{2+\sqrt{3}}\right)^x = 4.$

Solution:

It is easy to see that radical expressions are conjugate numbers. Consider the product of roots $\sqrt{2-\sqrt{3}}\cdot\sqrt{2+\sqrt{3}} = \sqrt{4-3} = 1.$

This makes it possible to express one term of the equation through the other: $\sqrt{2-\sqrt{3}} = \frac{1}{\sqrt{2+\sqrt{3}}}.$ Substituting it into the original equation, we get:

$\left(\frac{1}{\sqrt{2+\sqrt{3}}}\right)^x + \left(\sqrt{2+\sqrt{3}}\right)^x = 4.$ Let's introduce a new variable: $y = \left(\sqrt{2+\sqrt{3}}\right)^x, \quad y > 0;$

$\frac{1}{y} + y = 4.$ Multiplying both sides of the equation by y, we get: $y^2 - 4y + 1 = 0;$

$D = 16 - 4 = 12; \quad y_1 = \frac{4-\sqrt{12}}{2} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}; \quad y_2 = 2+\sqrt{3}.$ If $y = 2-\sqrt{3}$, then

$\left(\sqrt{2+\sqrt{3}}\right)^x = 2-\sqrt{3};\quad \left(2+\sqrt{3}\right)^{\frac{x}{2}} = 2+\sqrt{3};\quad \frac{x}{2}=1;\quad x=2.\quad \left(\sqrt{2+\sqrt{3}}\right)^x = \frac{1}{2+\sqrt{3}};$

$\left(\sqrt{2+\sqrt{3}}\right)^x = \left(2+\sqrt{3}\right)^{-1};\quad \left(2+\sqrt{3}\right)^{\frac{x}{2}} = \left(2+\sqrt{3}\right)^{-1};\quad \frac{x}{2}=-1;\quad x=-2.$

If $y=2+\sqrt{3}$, то $\left(\sqrt{2+\sqrt{3}}\right)^x = 2+\sqrt{3};\quad \left(2+\sqrt{3}\right)^{\frac{x}{2}} = 2+\sqrt{3};\quad \frac{x}{2}=1;\quad x=2.$

Answer: $-2;\ 2.$

Self-study assignments:

7.1 $3^x = 81.$ Answer: 4.

7.2 $3^{x^2-\frac{5}{7}x} = \sqrt[7]{9}.$ Answer: $-\frac{2}{7};\ 1.$

7.3 $\left(\frac{1}{9}\right)^x = \frac{1}{27}.$ Answer: 1,5.

7.4 $3^{2x+2} + 3^{2x} = 30.$ Answer: $\frac{1}{2}.$

7.5 $3^{6-x} = 5^{3x-2}.$ Answer: 2.

7.6 $3^{2x} - 6\cdot 5^x + 5 = 0.$ Answer: 0; 1.

7.7 $6^x = 1.$ Answer: 0.

7.8 $3\cdot 16^x + 2\cdot 81 = 5\cdot 36^x.$ Answer: 0; 0,5.

7.9 $e^x = 1.$ Answer: 0.

7.10 $(x+3)^{x^2-3} = (x+3)^{x^2}.$ Answer: $-2;\ -1;\ 3.$

7.11 $0{,}4^x = 2{,}5.$ Answer: $-4.$

7.12 $4^{x+\sqrt{x^2-2}} - 5\cdot 2^{x-1+\sqrt{x^2-2}} = 6.$ Answer: 1,5.

7.13 $\sqrt{2^x\cdot 3^x} = 36.$ Answer: 4.

7.14 $6^{2x+4} = 2^{x+8}\cdot 3^{3x}.$ Answer: 4.

7.15 $\sqrt{3^x} = 9.$ Answer: 4.

7.16 $\sqrt[x]{5^{5\sqrt{x}}} = 5^{\sqrt{x-4}} = 6.$ Answer: 25.

7.18 $2{,}56^{\sqrt{x}-1} = \left(\frac{5}{8}\right)^{4\sqrt{x}+1}.$ Answer: $\frac{1}{36}.$

7.19 $\left(x^2-x-1\right)^{x^2-1} = 1.$ Answer: $-1;\ 1;\ 2.$

7.20 $4^{x+1,5} + 2^{x+2} = 4.$ Answer: $-1.$

7.21 $3\cdot 4^x - 5\cdot 6^2 + 2\cdot 9^x = 0.$ Answer: 0; 1.

7.22 $3^{3x+1} - 4\cdot 27^{x-1} + 9^{1,5x-1} = 80.$ Answer: 1.

7.23 $0{,}25^{2-x} = \frac{256}{2^{x+3}}.$ Answer: $x=3.$

7.24 $7^{3^x} = \sqrt[9]{7}.$ Answer: $x=-2.$

7.25 $3^{2x} = 4 \cdot \sqrt[x]{9}$. Answer: $2;\ -\dfrac{\lg 3}{\lg 2}$.

7.26 $7^{1-x} = 7^{x-1}$. Answer: 1.

7.27 $8^x + 18^x - 2 \cdot 27^x = 0$. Answer: 0.

7.28 $4 \cdot 9^x - 7 \cdot 12^x + 3 \cdot 16^x = 0$. Answer: 0; 1.

7.29 $3^{2x-1} - 2 \cdot 15^x - 5^{2x+1} = 0$. Answer: -1.

7.30 $\left(\sqrt{7+\sqrt{48}}\right)^x + \left(\sqrt{7-\sqrt{48}}\right)^x = 14 \cdot \left(\sqrt{7+\sqrt{48}}\right)^x$. Answer: -2.

7.31 $3 \cdot 16^x + 2 \cdot 81^x - 5 \cdot 36^x = 0$. Answer: $\dfrac{1}{2}$; 0.

7.32 $5^x \cdot \sqrt[x]{8^{x-1}} = 500$. Answer: 3.

7.33 $5^{2x-3} = 2 \cdot 5^{x-2} + 3$. Answer: 2.

7.34 $(x-2)^{x^2-x} = (x-2)^{12}$. Answer: -3; 3; 4.

7.35 $\left(x^{x+y}\right) = x^{4.}$ Answer: 1.

7.36 $\dfrac{(0,5)^{x-3}}{(0,125)^{2-x}} = 128^x \cdot \sqrt[3]{(0,25)^{x-1}}$. Answer: $\dfrac{25}{31}$.

7.37 $\left(\dfrac{3}{2}\right)^{3x-1} \cdot \left(\dfrac{8}{27}\right)^{1-x} = \left(\dfrac{9}{4}\right)^x$. Answer: 1.

7.38 $4^{x+1} - 5^{x-1,5} = 5^{x+0,5} - 2^{2x-4}$. Answer: 2,5.

7.39 $\left(\sqrt[3]{\sqrt{48}-5}\right)^{2x} - \left(\sqrt[3]{\sqrt{26}+5}\right)^{2x} = 10$. Answer: $-1{,}5$.

7.40 $(1,4)^{x-8} \cdot \left(1\dfrac{11}{14}\right)^{x-8} = 6\dfrac{1}{4}$. Answer: 10.

7.41 $64^{tg^2 x} + 8 = 9 \cdot 8^{\frac{1}{\cos^2 x} - 1}$. Answer: $-\dfrac{\pi}{4};\ \dfrac{\pi}{4} + \pi n;\ \pi n;\ n \in Z$.

7.42 $x^{\log_2 x + 2} = 256$. Answer: $\dfrac{1}{16}$; 4.

7.43 $4^{x+1} + 4^{x-2} = 260$. Answer: 3.

7.44 $3 \cdot 16^x + 2 \cdot 81^x = 5 \cdot 36^x$. Answer: 0; 0,5.

7.45 $\left(2^{x-4}\right)^{x+3} = 0,5^x \cdot 4^{x+6}$. Answer: -4; 16.

7.46 $3^{x+3} + 5 \cdot 3^{x+2} - 8 \cdot 3^{x+1} = 1\dfrac{7}{9}$. Answer: -3.

7.47 $4 \cdot 9^x - 7 \cdot 12^x + 3 \cdot 16^x = 0$. Answer: $\{0;\ 1\}$.

7.48 $64 \cdot 9^x - 84 \cdot 12^x + 27 \cdot 16^x = 0$. Answer: $\{1;\ 2\}$.

7.49 $5 \cdot 4^x - 7 \cdot 10^x + 2 \cdot 25^x = 0$. Answer: $\{0;\ 1\}$.

7.50 $7 \cdot 4^{x^2} - 9 \cdot 14^{x^2} + 2 \cdot 49^{x^2} = 0$. Answer: $\{-1;\ 0;\ 1\}$.

7.51 $4^{x+1,5} + 9^x = 6^{x+1}$. Answer: $\left\{\log_{\frac{2}{3}} \dfrac{1}{4};\ \log_{\frac{2}{5}} \dfrac{1}{2}\right\}$.

7.52 $3 \cdot 16^x + 37 \cdot 36^x = 26 \cdot 81^x$. Answer: $\{0,5\}$.