Section 15
Exponential inequalities

Inequalities containing a variable in the exponent are called *exponential.*
Solution of exponential inequalities for $a > 0, a + 1$ are based on the monotonicity of the exponential function, i.e.:
1). If $0 < a < 1,$ then from the inequality $a^{x_1} > a^{x_2}$ it follows that $x_1 < x_2$.
2). If $a > 1,$ then from the inequality $a^{x_1} > a^{x_2}$ follows that $x_1 > x_2$.
Let's demonstrate this with examples.:

$2^x < \frac{1}{4}.$

Solution:

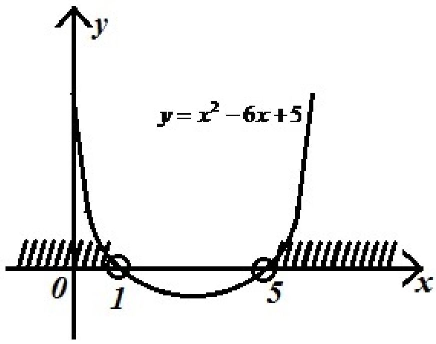
$2^x < \frac{1}{2^2};\ 2^x < 2^{-2}.$ Insofar as $2 > 1,$ то $x < -2.$

Answer: $(-\infty;-2).$

$(0{,}36)^{-x^2+6x} > (0{,}36)^5.$

Solution:

Because $0 < 0{,}36 < 1,$ то $-x^2 + 6x < 5 \,|\cdot(-1)\ \ x^2 - 6x + 5 > 0,\ \ x_1 = 1;\ \ x_2 = 5.$

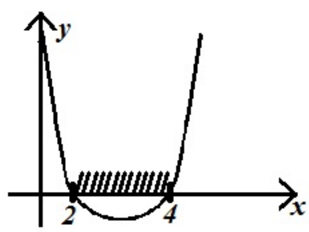


Answer: $(-\infty;1)\cup(5;+\infty).$

$4^x - 6\cdot 2^x + 8 < 0.$

Solution:

$(2^x)^2 - 6\cdot 2^x + 8 < 0;$ By Vieta's theorem, we have:



$\left[\begin{array}{l} 2^x = 2, \\ 2^x = 4. \end{array}\right. \left[\begin{array}{l} x = 1, \\ x = 2. \end{array}\right.$

$2 < 2^x < 4,$
$2^1 < 2^x < 2^2,$
$1 < x < 2.$

Answer: $(1;2).$

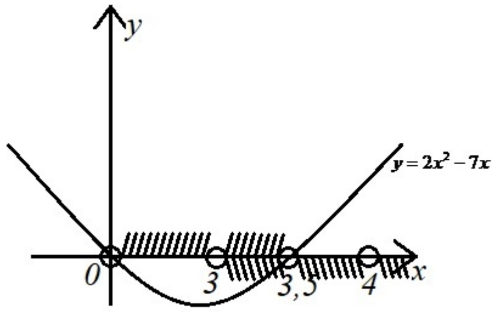
$(x-3)^{2x^2-7x} > 1.$

Solution:

We can consider two cases:

1). $0 < x - 3 < 1,\ (x-3)^{2x^2-7x} > (x-3)^0,\ 2x^2 - 7x < 0.$

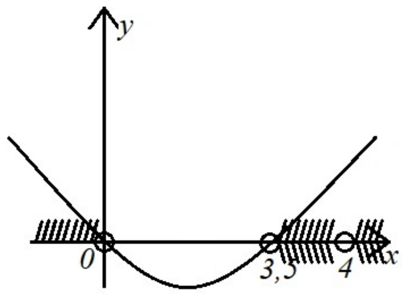
$$\begin{cases} 0 < x - 3 < 1 \,\big|\, +3 \\ 2x^2 - 7x < 0. \end{cases} \quad \begin{cases} 3 < x < 4, \\ x(2x-7) < 0. \end{cases}$$



$x \in (3;\, 3{,}5).$

2). $x - 3 > 1,\ x > 4.\ (x-3)^{2x^2-7x} > (x-3)^0,\ 2x^2 - 7x > 0.$

$$\begin{cases} x > 4, \\ 2x^2 - 7x > 0. \end{cases}$$



$(4;+\infty).$

Answer: $(3;\, 3{,}5) \cup (4;+\infty).$

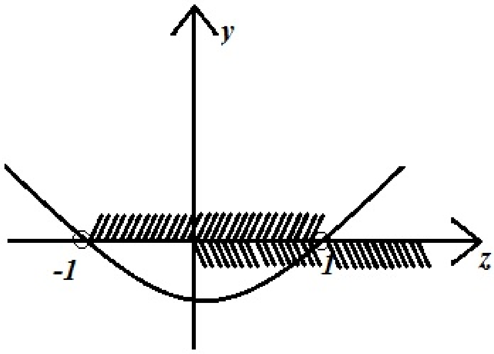
$4^x - 2 \cdot 5^{2x} < 10^x.$

Solution:

$4^x - 2 \cdot 5^{2x} < 10^x \,\big|\, : 10^x,\ \dfrac{4^x}{10^x} - \dfrac{2 \cdot 25^x}{10^x} < \dfrac{10^x}{10^x};\ \left(\dfrac{2}{5}\right)^x - 2 \cdot \left(\dfrac{5}{2}\right)^x < 1;$

We denote $z = \left(\dfrac{2}{5}\right)^x, z > 0.\ z - \dfrac{2}{z} - 1 < 0,\ \dfrac{z^2 - z - 2}{z} < 0.$

Insofar as $z > 0,\ z^2 - z - 2 < 0,\ z^2 - z - 2 = 0,\ z_1 = 2,\ z_2 = -1.$



$a < \left(\frac{2}{5}\right)^x < 2;\ \ 0 < (0{,}4)^x < 2;\ \ x\log_{0,4} 0{,}4 > \log_{0,4} 2;\ \ x > \log_{0,4} 2.$

Answer: $(\log_{0,4} 2; +\infty)$.

$2^{3x-1} < \sqrt[4]{8}.$

Solution:

$2^{3x-1} < 2^{\frac{3}{4}};\ \ 2 > 1,\ 3x-1 < \frac{3}{4};\ \ 3x < 1\frac{3}{4};\ \ 3x < \frac{7}{4} \Big| :3,\ \ x < \frac{7}{12}.$

Answer: $\left(-\infty; \frac{7}{12}\right)$.

$\frac{15}{2^x+1} + \frac{4}{2^{x-1}-3} > \frac{12}{2^{x+1}}.$

Solution:

let the $2^x = t,\ t > 0,$ then $\frac{15}{t+1} + \frac{4}{\frac{t}{2}-3} > \frac{12}{t\cdot 2};\ \ \frac{15}{t+1} + \frac{8}{t-6} - \frac{12}{2t} > 0;\ \ \frac{15}{t+1} + \frac{8}{t-6} - \frac{6}{t} > 0;$

$\frac{15t\cdot(t-6) + 8t\cdot(t+1) - 6\cdot(t+1)\cdot(t-6)}{(t-6)\cdot t\cdot(t+1)} > 0;\ \ \frac{15t^2 - 90t + 8t^2 + 8t - 6t^2 + 36t - 6t + 36}{(t-6)\cdot t\cdot(t+1)} > 0;$

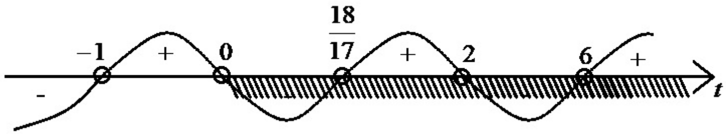
$\frac{17t^2 - 52t + 36}{(t-6)\cdot t\cdot(t+1)} > 0.$

We expand the numerator factoring $D = 2704 - 2448 = 256 = 16^2.$

$t_1 = \frac{52-16}{34} = \frac{36}{34} = \frac{18}{17};\ \ t_2 = \frac{52+16}{34} = \frac{68}{34} = 2.$

$\frac{14\cdot\left(t - \frac{18}{17}\right)\cdot(t-2)}{(t-6)\cdot t\cdot(t+1)} > 0.$

We solve this inequality by the "snake" method:



$t \in (-1; 0) \cup \left(\frac{18}{17}; 2\right) \cup (6; +\infty).$

Considering that $t > 0,$ we get:

$t \in \left(\frac{18}{17}; 2\right) \cup (6; +\infty).$

Let's go back to the replacement $t=2^x$, $\frac{18}{17}<2^x<2$ or $2^x>6$.

$\log_2\frac{18}{17}<x\log_2 2<\log_2 2,\ \log_2\frac{18}{17}<x<1.$

$\log_x 2^x>\log_2 6,\ x\log_2 2>\log_2 6,\ x>\log_2 6.$

Answer: $\left(\log_2\frac{18}{17};\ 1\right)\cup(\log_2 6;\ +\infty).$

$3\cdot\left(x^2\cdot 2^{\sqrt{x-2}}+x+2\right)<9x^2+x\cdot 2^{\sqrt{x-2}}+2^{\sqrt{x-2}+1}.$

Solution:

$3x^2\cdot 2^{\sqrt{x-2}}+3x+6<9x^2+x\cdot 2^{\sqrt{x-2}}+2^{\sqrt{x-2}+1};$

$3x^2\cdot 2^{\sqrt{x-2}}-x\cdot 2^{\sqrt{x-2}}-2^{\sqrt{x-2}+1}\cdot 2+3x+6-9x^2<0;$

$2^{\sqrt{x-2}}\cdot\left(3x^2-x-\cdot 2\right)-3\cdot\left(3x^2-x-2\right)<0;$

$\left(3x^2-x-\cdot 2\right)\cdot\left(2^{\sqrt{x-2}}-3\right)<0;$ This inequality is equivalent to such a set:

$$\left[\begin{array}{l}\begin{cases}3x^2-x-2<0, & D=25,\ x_1=-\frac{2}{3};\ x_2=1,\\ 2^{\sqrt{x-2}}-3>0. & x-2\ge 0,\ x\ge 2.\end{cases}\quad \begin{cases}x\in\left(-\frac{2}{3};\ 1\right)\\ x\in[2;+\infty)\end{cases}\quad \varnothing\\ \begin{cases}3x^2-x-2>0,\\ 2^{\sqrt{x-2}}-3<0.\end{cases}\quad \begin{cases}x\in\left(-\infty;-\frac{2}{3}\right)\cup(1;+\infty),\\ 2^{\sqrt{x-2}}<3,\ x\ge 2.\end{cases}\end{array}\right.$$

$\log_2 2^{\sqrt{x-2}}<\log_2 3,\ \sqrt{x-2}\log_2 2<\log_2 3,\ \sqrt{x-2}<\log_2 3.$

Let us square both sides of the last inequality:

$0<x-2<\log_2^2 3\ \left|+2\right.\qquad 2<x<\log_2^2 3+2.$

Answer: $\left(2;\ 2+\log_2^2 3\right).$

$\left(2x^2+x+1\right)^{2x^2+x-1}<1.$

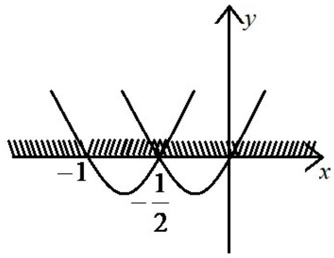
Solution:

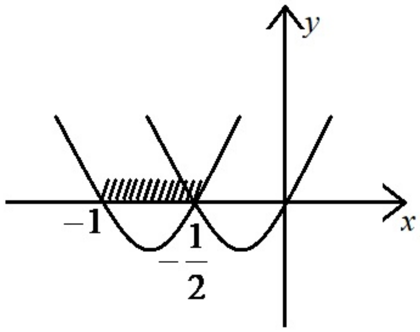
Expression $2x^2+x+1>0$, бо $D=1-8=-7<0$ and the branches of the parabola are directed upwards.

This makes it possible to replace with one: $1=\left(2x^2+x+1\right)^0$.

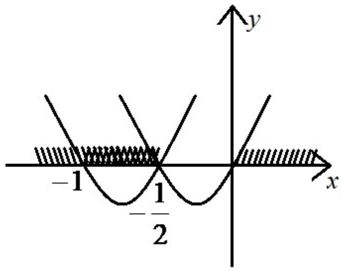
$\left(2x^2+x+1\right)^{2x^2+x-1}<\left(2x^2+x+1\right)^0;$

$$\left[\begin{array}{l}\begin{cases}2x^2+x+1<1,\\ 2x^2+x-1>0.\end{cases}\quad\begin{cases}2x^2+x<1,\\ D=1+8=9.\end{cases}\\ \begin{cases}2x^2+x+1>1,\\ 2x^2+x-1<0.\end{cases}\quad\begin{cases}x(2x+1)>0,\\ \end{cases}\end{array}\right.\quad\begin{cases}x(2x+1)<0,\\ x_1=\frac{-1-3}{4}=-1;\ x_2=-\frac{1}{2}.\end{cases}$$





$x \in (1;+\infty)$



$x \in \left(-1;-\frac{1}{2}\right)$.

Answer: $\left(-1;-\frac{1}{2}\right) \cup (1;+\infty)$.

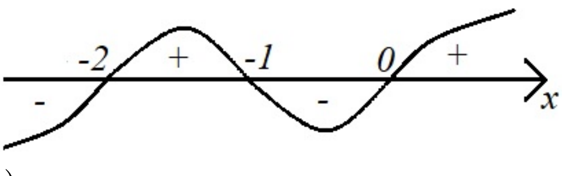
$\sqrt[x+1]{0{,}25^x} > 4^x$.

Solution:

$\sqrt[x+1]{\left(\frac{25}{100}\right)^x} > 4^x$, $\sqrt[x+1]{\left(\frac{1}{4}\right)^x} > 4^x$, $\sqrt[x+1]{\left(4^{-1}\right)^x} > 4^x$, $\sqrt[x+1]{4^{-x}} > 4^x$, $4^{-\frac{x}{x+1}} > 4^x$. Because $4 > 1$, то $-\frac{x}{x+1} > x$, $-\frac{x}{x+1} - x > 0, \left| \cdot (-1) \right.$ $\frac{x}{x+1} + x < 0$, $\frac{x + x^2 + x}{x+1} < 0$, $\frac{x^2 + 2x}{x+1} < 0 \left| \cdot (x+1)^2 \right.$, бо $(x+1)^2 > 0$, $\quad x + 1 \neq 0$.

Then we obtain an inequality equivalent to this inequality in the domain of definition:

$(x+1) \cdot (x^2 + 2) < 0$, $(x+1) \cdot x \cdot (x+2) < 0$.



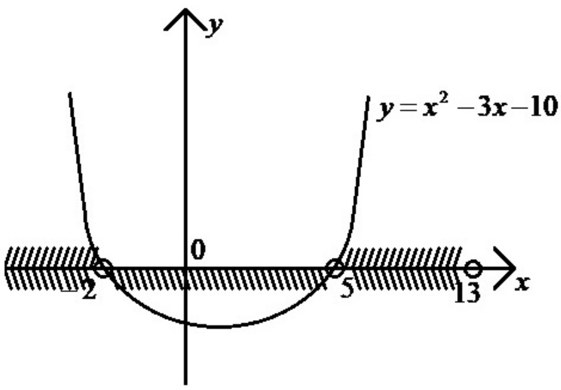
Answer: $(-\infty;-2) \cup (-1;0)$.

Logarithmic inequalities

An inequality that contains a variable under the sign of the logarithm is called *logarithmic.*
When solving logarithmic inequalities, it is necessary to take into account the following two factors:
1). Monotonicity of a logarithmic function;
2). The domain of the logarithmic function is the set of positive numbers.
$\log_2\left(x^2-4x+3\right)>\log_2(13-x)$.

Solution:

This inequality is equivalent to such a system of inequalities:
$\begin{cases} x^2-4x+3>13-x, \\ 13-x>0. \end{cases}$ $\begin{cases} x^2-4x+3-13+x>0, \\ x<13. \end{cases}$ $\begin{cases} x^2-3x+-10>0, \\ x<13. \end{cases}$
$x_1=5,\ x_2=-2.$



Answer: $(-\infty;-2)\cup(5;13)$.
$\frac{1}{2}\lg(x-1)+\lg\sqrt{2x+1}<1+\lg 0{,}3.$

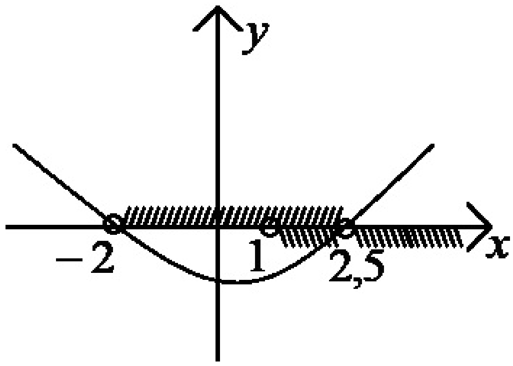
Solution:

$\lg\sqrt{x-1}+\lg\sqrt{2x+1}<\lg 10+\lg 3-\lg 10;$
$\lg\sqrt{(x-1)(2x+1)}<\lg 3;$
Because the $10>3$, then, since the logarithmic function is monotonic, we have: $\sqrt{(x-1)(2x+1)}<3$. After squaring, the inequality is formed: $(x-1)(2x+1)<9$. Consider the system of inequalities:

$\begin{cases} x-1>0, \\ 2x+1>0, \\ (x-1)\cdot(2x+1)<9 \end{cases}$ $\begin{cases} x>1, \\ x>\frac{1}{2}, \\ 2x^2+x-2x-1-9<0 \end{cases}$ $\begin{cases} x>1, \\ 2x^2-x-10<0. \end{cases}$

$D=1+80=81,\ x_1=\frac{1-9}{4}=-2-$ does not satisfy the condition $x>1$.

$x_1=\frac{1+9}{4}=2{,}5>1.$



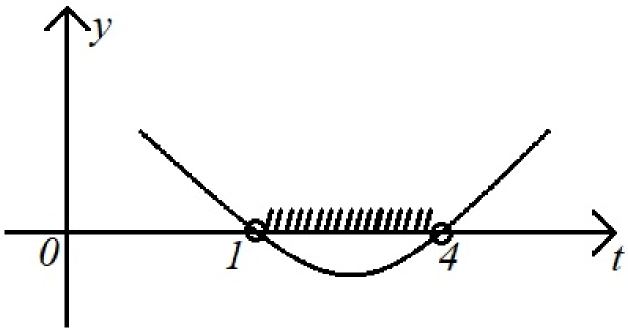
Answer: $(1;\ 2{,}5)$.

$\log_{\frac{1}{2}} x + 5\log_2 x + 4 < 0.$

Solution:

$\log^2_{\frac{1}{2}} x + 5\cdot\dfrac{\log_{\frac{1}{2}} x}{\log_{\frac{1}{2}} 2} + 4 < 0,\ \ \log^2_{\frac{1}{2}} x + 5\cdot\dfrac{\log_{\frac{1}{2}} x}{-1} + 4 < 0,\ \ \log^2_{\frac{1}{2}} x - 5\cdot\log_{\frac{1}{2}} x + 4 < 0.$

Let the $\log_{\frac{1}{2}} x = t,$ then $t^2 - 5t + 4 < 0,\ t_1 = 1,\ t_2 = 4.$



$1 < \log_{\frac{1}{2}} x < 4,\ \ 1 < \log_{\frac{1}{2}} \dfrac{1}{2} < \log_{\frac{1}{2}} x < \log_{\frac{1}{2}} \dfrac{1}{16},\ \ x > 0.$

Because the $0 < \dfrac{1}{2} < 1,$ then, due to the monotonicity of the logarithmic function, we have $\dfrac{1}{2} > x > \dfrac{1}{16},$ or $\dfrac{1}{16} < x < \dfrac{1}{2}.$

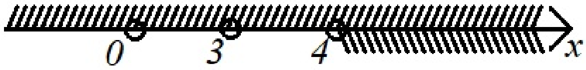
Answer: $\left(\dfrac{1}{16};\ \dfrac{1}{2}\right).$

$\dfrac{\log_a x^2}{\log_a (x-3)} < 2$ at $a > 1.$

Solution:

$\dfrac{\log_a x^2}{\log_a (x-3)} - -2 < 0.$ Let us find the domain of definition of the left side of this inequality.

$\begin{cases} x^2 > 0, \\ x - 3 > 0, \\ x - 3 \neq 1. \end{cases} \begin{cases} x \in (-\infty; 0) \cup (0; +\infty), \\ x \in (3; +\infty), \\ x \neq 4. \end{cases}$



The domain of this function $(3;4)\cup(4;+\infty)$.

We denote $f(x)=\dfrac{\log_a x^2}{\log_a(x-3)}-2.$

Let us find the zeros of this function by solving the following equation:

$\dfrac{\log_a x^2}{\log_a(x-3)}-2=0;$ By condition $a>1,$ therefore can be reduced to a common denominator:

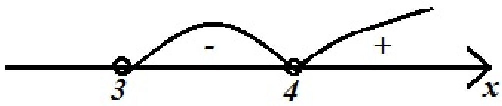
$\dfrac{\log_a x^2-2\log_a(x-3)}{\log_a(x-3)}=0;$ $\begin{cases}\log_a x^2-\log_a(x-3)=0,\\ \log_a(x-3)\neq 0.\end{cases}$ $\log_a x^2=\log_a(x-3)^2.$

Since the logarithmic function is monotonic, we have: $x^2=(x-3)^2;$ $x^2=x^2-6x+9,$

$x^2-x^2+6x=9,$ $6x=9,$ $x=\dfrac{9}{6}=\dfrac{3}{2}.$

$\dfrac{3}{2}\notin D(f).$ Hence the function $f(x)$ has no roots, so it preserves its sign on each of the intervals belonging to its domain of definition.

Let us define the sign of the function f at each of the intervals:



$$f(3{,}5)=\frac{\log_a 3{,}5}{\log_a(3{,}5-3)}-2=\frac{\log_a 3{,}5}{\log_a(3{,}5-3)}-2=\frac{2\log_a 3{,}5-2\log_a 0{,}5}{\log_a 0{,}5}=\frac{\log_a 3{,}5^2-\log_a 0{,}5^2}{\log_a 0{,}5}=$$

$$=\frac{\log_a \dfrac{12{,}25}{0{,}25}}{\log_a 0{,}5}=\frac{\log_a 49}{\log_a 0{,}5}<0;$$

$$f(10)=\frac{\log_a 10^2}{\log_a(10-3)}-2=\frac{\log_a 100-2\log_a 7}{\log_a 7}=\frac{\log_a \dfrac{100}{49}}{\log_a 7}=\frac{\log_a 2\dfrac{2}{49}}{\log_a 7}>0.$$

Answer: $(3;4).$

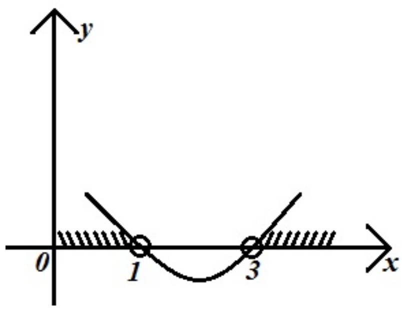
$\log_{(x-3)}(x^2-4x+3)<0.$

Solution:

Let the $f(x)=\log_{(x-3)}(x^2-4x+3).$ We find the domain of the function:

$f(x)$: $x^2-4x+3>0,$ $x_1=1;$ $x_2=3-$ the roots of quadratic polynomial.

$D(f)=(-\infty;-1)\cup(3;+\infty).$



$\log_{(x-3)}\left(x^2-4x+3\right)<\log_{(x-3)}1.$

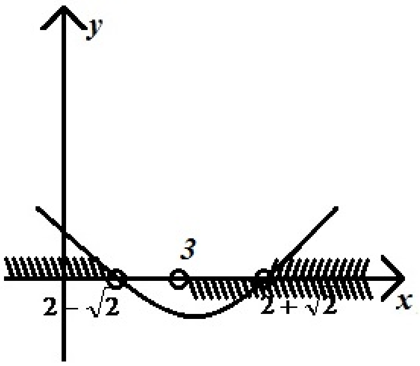
Solution:

This inequality is equivalent to such a set of systems of inequalities:

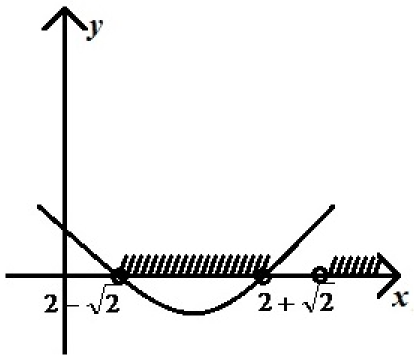
$$\left[\begin{array}{l}\begin{cases}x^2-4x+3>1,\\0<x-3<1.\end{cases}\\ \begin{cases}x^2-4x+3<1,\\x-3>1.\end{cases}\end{array}\right.\quad\left[\begin{array}{l}\begin{cases}x^2-4x+2>0,\\3<x<4.\end{cases}\\ \begin{cases}x^2-4x+2<0,\\x>4.\end{cases}\end{array}\right.$$

To solve these systems, we use a graphic illustration:

$D=16-8=8=4\cdot 2;\ \ x_1=\dfrac{4-\sqrt{4\cdot 2}}{2}=\dfrac{4-2\sqrt{2}}{2}=2-2\sqrt{2};\ \ x_2=2+2\sqrt{2}.$



$x\in\left(2+\sqrt{2};4\right)\in D(f)0.$



$\varnothing$

Answer: $\left(2+\sqrt{2};4\right)$.

$\log_x(2{,}5-x)<-1.$

Solution:

$f(x)=\log_x(2{,}5-x),\ \ 2{,}5-x>0,\ \ x<2{,}5.$

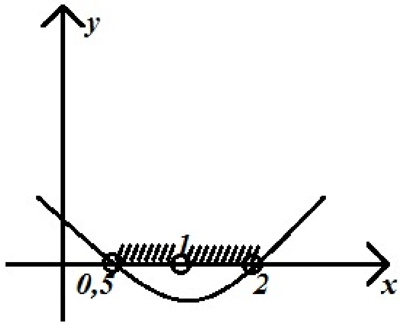
$x>0,\ \ x\neq 1.\ \ \begin{cases}0<x<2{,}5,\\x\neq 1.\end{cases}\ \ D(f)=(0;1)\cup(1;2{,}5).$

$-1=\log_x x^{-1}=\log_x\dfrac{1}{x};$ Then $\log_x(2{,}5-x)<\log_x\dfrac{1}{2}.$

Due to the monotonicity of the logarithmic function at $0<x<1,\ \ 2{,}5-x>\dfrac{1}{x},$

$2{,}5-x-\dfrac{1}{x}>0,\ \ x^2-2{,}5x+1<0,\ \ D=6{,}25-4=2{,}25=1{,}5^2.$

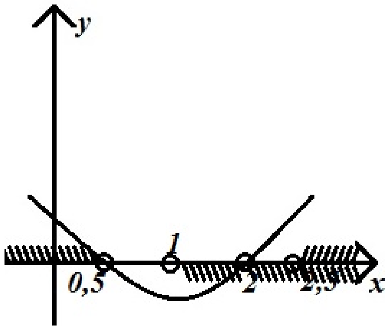
$x_1=\dfrac{2{,}5-1{,}5}{2}=0{,}5;\ \ x_2=\dfrac{2{,}5+1{,}5}{2}=2.$



$x \in (0{,}5;1)$.

At $x > 1,\ 2{,}5 - x < \dfrac{1}{2},\ \ -x^2 + 2{,}5x - 1 < 0 \,\big|\cdot(-1)$

$x^2 - 2{,}5x + 1 > 0.$



$x \in (2;2{,}5)$.

Answer: $(0{,}5;1) \cup (2;2{,}5)$.

$x^{\log_2 x + 1} > 8x.$

Solution:

Let us logarithm both sides of the inequality with the basis 2:

Range of valid values: $(0;+\infty)$

$\log_2\left(x^{\log_2 x+1}\right) > \log_2(8x);$

$(\log_2 x + 1)\log_2 x > \log_2 8 + \log_2 x;$

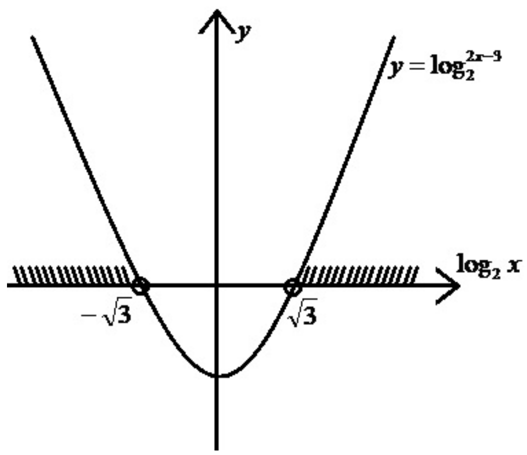
$\log_2^2 x + \log_2 x > \log_2 2^3 + \log_2 x;$

$\log_2^2 x + \log_2 x > 3 + \log_2 x;$

$\log_2^2 x - 3 > 0;$

$\log_2^2 x - \left(\sqrt{3}\right)^2 > 0;\ \ \left(\log_2 x - \sqrt{3}\right)\cdot\left(\log_2 x + \sqrt{3}\right) > 0;$

$$\left[\begin{array}{l}\log_2 x = \sqrt{3},\\ \log_2 x = -\sqrt{3}.\end{array}\right.$$



$-\infty < \log_2 x < -\sqrt{3}, \quad \sqrt{3} < \log_2 x < \infty, \quad 0 < x < 2^{-\sqrt{3}}, \quad 2^{\sqrt{3}} < x < \infty.$

Answer: $\left(0; \frac{1}{2^{\sqrt{3}}}\right) \cup \left(2^{\sqrt{3}}; +\infty\right).$

Systems of exponential inequalities

$$\left(\frac{8}{9}\right)^{-x} = \left(\frac{2}{3}\right)^{-x} = \left(\left(\frac{2}{3}\right)^{2}\right)^{-x} \cdot 2^{-x} = \left(\left(\frac{2}{3}\right)^{-x}\right)^{2} \cdot \frac{1}{2^{x}}.$$

$$\underbrace{\left(\frac{2}{3}\right)^{x} \cdot \left(\frac{2}{3}\right)^{-x}}_{1} \cdot \left(\frac{2}{3}\right)^{-x} \cdot \frac{1}{2^{x}} > \frac{3^3}{4^3}; \quad \left(\frac{3}{2}\right)^{x} \cdot \frac{1}{2^{x}} > \left(\frac{3}{4}\right)^{3}.$$

$$\frac{3^x}{2^x \cdot 2^x} > \left(\frac{3}{4}\right)^{3}; \quad \frac{3^x}{2^{2x}} > \left(\frac{3}{4}\right)^{3}; \quad \frac{3^x}{4^x} > \left(\frac{3}{4}\right)^{3}; \quad \left(\frac{3}{4}\right)^{x} > \left(\frac{3}{4}\right)^{3}.$$

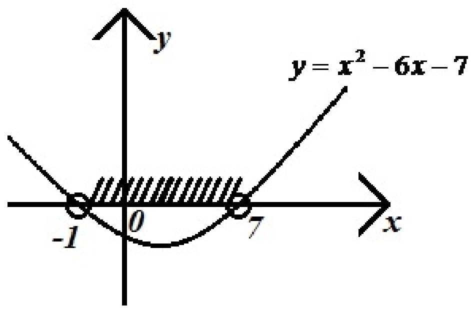
Because $0 < \frac{3}{4} < 1,$ then, since the exponential function is monotonic, we have:

$x < 3.$

We solve the second inequality of the system:

$2^{x^2-6x-3,5} < 2^3 \cdot 2^{\frac{1}{2}}; \quad 2^{x^2-6x-3,5} < 2^{3\frac{1}{2}}; \quad 2 > 1,$ and therefore $x^2 - 6x - 7 < 0, \quad x^2 - 6x - 7 = 0.$

By Vieta's theorem $x_1 = -1, \quad x_2 = 7.$



Answer: $(-1;3).$

$1 < 3^{\left|x^2 - x\right|} < 9.$

Solution:

$3^0 < 3^{\left|x^2 - x\right|} < 3^2;$

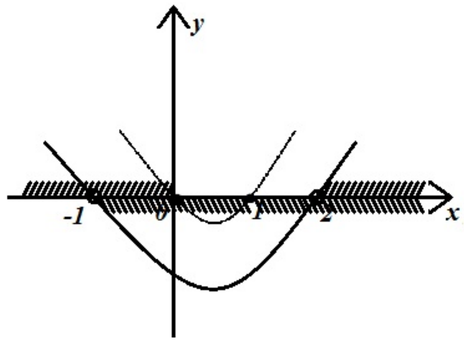
$0 < \left|x^2 - x\right| < 2.$

Due to the monotony of the exponential function have:

$x^2 - x = 0, \quad x(x-1) = 0, \quad x = 0, \quad x = 1.$

Ha $(-\infty;0);$ $\left|x^2 - x\right| = x^2 - x.$

$$0 < x^2 - x < 2 \rightarrow \begin{cases} x^2 - x > 0, \\ x - x < 2. \end{cases} \quad \begin{cases} x(x-1) > 0, \\ x^2 - x - 2 < 0. \end{cases}$$



$x \in (-1;0)$

On $[0;1)$: $|x^2 - x| = -(x^2 - x)$, $0 < -(x^2 - x) < 1 | \cdot (-1)$

$$-1 < x^2 - x < 0 \rightarrow \begin{cases} x^2 - x > -1, & x^2 - x + 1 < 0, \\ x^2 - x < 0. & D < 0. \end{cases} \quad \varnothing.$$

Ha $[1;+\infty)$: $|x^2 - x| = x^2 - x$, $x \in [1;2)$.

Answer: $(-1;0) \cup [1;2)$.

$$\begin{cases} \left(\frac{2}{3}\right)^x \cdot \left(\frac{8}{9}\right)^{-x} > \frac{27}{64}, \\ 2^{x^2 - 6x - 3,5} < 8\sqrt{2}. \end{cases}$$

<u>Systems of logarithmic inequalities</u>

$$\frac{\lg 7 - \lg(-8x - x^2)}{\lg(x+3)} > 0.$$

Solution:

This inequality is equivalent to the combination of two systems of inequalities:

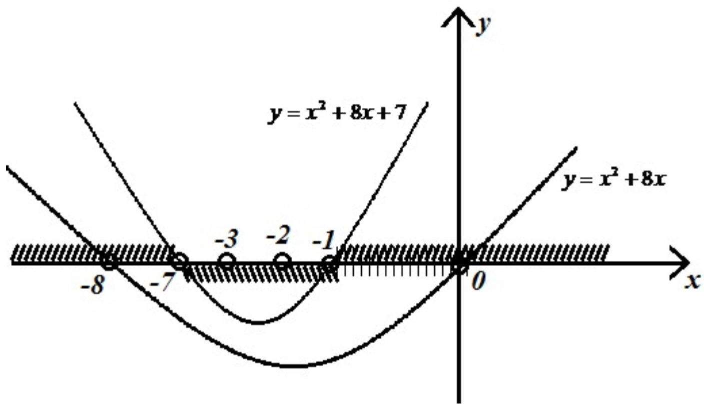
$$\left[\begin{array}{l} \begin{cases} \lg 7 - \lg(-8x - x^2) > 0, \\ \lg(x+3) > 0. \end{cases} \\ \begin{cases} \lg 7 - \lg(-8x - x^2) < 0, \\ \lg(x+3) < 0. \end{cases} \end{array} \right.$$

Let's solve each of these systems in particular.

$$\begin{cases} \lg 7 > \lg(-8x - x^2), \\ \lg(x+3) > \lg 1, \\ -8x - x^2 > 0 | \cdot (-1) \\ x + 3 > 0. \end{cases} \begin{cases} 7 > -8x - x^2, \\ x + 3 > 1, \\ x^2 + 8x < 0, \\ x > -3. \end{cases} \begin{cases} x^2 + 8x + 7 > 0, \\ x > -2, \\ x(x+8) < 0, \\ x > -3. \end{cases}$$

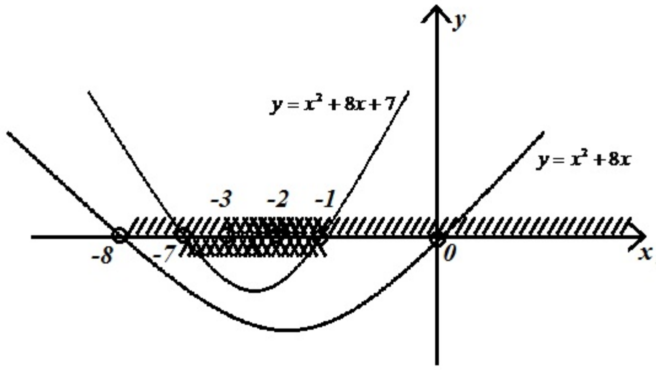
$x_1 = -1$, $x_2 = -7$ (Vieta's theorem).

$x_3 = 0$, $x_4 = -8$.



$x \in (-1;0)-$ Solution of the first system.

$$\begin{cases} \lg 7 < \lg(-8x - x^2), \\ \lg(x+3) < \lg 1, \\ -8x - x^2 > 0, \\ x+3 > 0. \end{cases} \quad \begin{cases} 7 < -8x - x^2, \\ x+3 < 1, \\ x^2 + 8x < 0, \\ x+3 > 0. \end{cases} \quad \begin{cases} x^2 + 8x - 7 < 0, \\ x < -2, \\ x(x+8) < 0, \\ x > -3. \end{cases}$$



$x \in (-3;-2)$.

Answer: $(-3;-2) \cup (-1;0)$.

Self-study assignments:

Prove that for all values of *x* the correct inequality:

1). $x + 3(x+1) - 4(x+2) < 0$,

2). $(x-2)^2 - x(x-4) > 0$,

3). $(5a+1) \cdot (5a-1) < 25a^2$,

4). $\dfrac{(1+x)^2}{2} \ge 2x$,

5). $\dfrac{1}{m} + m \ge 2$ at $m > 0$.

6). $x^2 + 1 \ge 2x$,

7). $x^2 + y^2 + z^2 \ge xy + xz + yz$.

8). $(a+1) \cdot (b+1) \cdot (a+c) \cdot (b+c) > 16abc$, at $a > 1, b > 1, c > 1$.

Solve inequalities:

1). $\frac{5-3x}{2}+4>\frac{x-7}{3}-x.$ Answer: $x\in\left(-\infty;\frac{53}{5}\right).$

2). $2x^2+3x-2>0.$ Answer: $(-\infty;-2)$ i $\left(\frac{1}{2};+\infty\right).$

3). $-3x+5x-4>0.$ Answer: $\varnothing.$

4). $x^4-42x^2-64x+105<0.$ Answer: $x\in(-5;-3)\cup(1;7).$

5). $\frac{x^3-3x^2+2x}{x^2+x-20}<0.$ Answer: $x\in(-\infty;-5)\cup(0;1)\cup(2;4).$

6). $x-\frac{x-1}{2}>\frac{x-3}{4}-\frac{x-2}{3}.$ Answer: $x\in(-1;+\infty).$

7). $x^2-6x+8>0.$ Answer: $x\in(-\infty;2)\cup(4;+\infty).$

8). $x^2+x-6\le 0.$ Answer: $x\in[-3;2].$

9). $x^2+6x+15<0.$ Answer: $\varnothing.$

10). $x^2-8x+16>0.$ Answer: $x\in(-\infty;4)\cup(4;+\infty).$

11). $x^2+10x+30<0.$ Answer: $x\in(-\infty;\infty).$

12). $(x+2)\cdot(x-3)\cdot(x-5)<0.$ Answer: $x\in(-\infty;2)\cup(3;5).$

13). $(x+1)\cdot(x-1)\cdot(x-2)\cdot(x-3)\cdot(x-5)>0.$ Answer: $x\in(-1;1)\cup(2;3)\cup(5;+\infty).$

14). $x^3-6x^2+11x-6\le 0.$ Answer: $x\in(-\infty;1]\cup[2;3].$

15). $\frac{x^2+5x+4}{x^2-6x+5}<0.$ Answer: $(-4;-1)$ i $(1;5).$

16). $2-\frac{x-3}{x-2}>\frac{x-5}{x-1}.$ Answer: $x\in(1;1{,}8)\cup(2;+\infty).$

17). $x^4-x^3-3x^2+4x-4\le 0.$ Answer: $[-2;2].$

18). $(x+4)\cdot(x-2)\cdot(x-3)\cdot(x+1)\le 0.$ Answer: $[-4;1]\cup[2;3].$

19). $x^3\cdot(x+3)^2\cdot(5+x)\cdot(2x-1)^4\cdot(2-x)<0.$ Answer: $(-5;-3)\cup(-3;0)\cup(2;+\infty).$

20). $(x-3)^5\cdot(x+1)^6\cdot(x+3)\cdot x^2\cdot(1-x)\ge 0.$ Answer: $(-\infty;-3]\cup\{-1;0\}\cup[1;3].$

21). $(x+3)\cdot(x-2)^2\cdot(x^2-2x+3)>0.$ Answer: $(-3;2)\cup(2;+\infty).$

22). $(x+2)^2\cdot(x^2+2x+4)\cdot(x-1)^2\cdot(x-2)<0.$ Answer: $(-2;1)\cup(1;2).$

23). $(x^2+4x+10)^2-7\cdot(x^2+4x+11)+7<0.$ Answer: $(-3;-1).$

24). $(x^2+x+1)\cdot(x^2+x+2)\le 12.$ Answer: $(-2;1).$

25). $\frac{(x-3)\cdot(x+2)\cdot(x-5)}{3x+4}\le 0.$ Answer: $\left[-2;-1\frac{1}{3}\right]\cup[3;5].$

26). $\frac{(x+1)\cdot(x-2)^3}{(x+3)^2}\ge 0.$ Answer: $(-\infty;-3)\cup(-3;-1]\cup[2;+\infty).$

27). $\frac{(x-2)^2\cdot(x+3)^3\cdot(x-5)}{x^4\cdot(1-x)\cdot(2x+9)}>0.$ Answer: $(-4{,}5;-3)\cup(1;2)\cup(2;5).$

28). $\frac{(x-4)^6\cdot(x+2)^5}{3-2x}<0.$ Answer: $(-\infty;-2)\cup(1{,}5;4)\cup(4;+\infty).$

29). $\frac{(x+2)\cdot(x-1)^2\cdot(x+3)}{x^2-4}\le 0.$ Answer: $[-3;2)\cup(-2;2).$

30). $\dfrac{(x^2-4x+3)\cdot(2x-x^2-5)}{(x^2-6x+5)\cdot(x^2+6x+8)}\le 0.$ Answer: $(-\infty;-4)\cup(-2;1)\cup(1;3]\cup(5;+\infty).$

31). $\dfrac{x^3+3x^2-x-3}{x^2+3x-10}<0.$ Answer: $(-\infty;-5)\cup(-3;-1)\cup(1;2).$

32). $\dfrac{(x-1)\cdot(x-2)\cdot(x-3)}{(x+1)\cdot(x+2)\cdot(x+3)}\le 1.$ Answer: $(-3;-2)\cup(-1;+\infty).$

33). $\sqrt{x^2+2x-15}<\sqrt{x^2+x-10}.$ Answer: $(-\infty;-5]\cup[3;5).$

34). $\sqrt[6]{2x^3-6x^2-10x+4}>\sqrt[6]{x^3-5x^2-6x}.$ Answer: $[-1;1)\cup[6;+\infty).$

35). $(x-1)\cdot(x^2-2x)\cdot\sqrt{16-x^2}>0.$ Answer: $[0;1]\cup[2;4].$

36). $\dfrac{x^2+x-12}{\sqrt{x^2+x-6}}\le 0.$ Answer: $[-4;3)\cup(2;3).$

37). $\dfrac{\sqrt{17-5x-2x^2}}{x+3}\ge 0.$ Answer: $(-3;1].$

38). $\log_{\sqrt{\frac{2x+3}{x-1}}}\left(\sqrt{6}+\sqrt{5}\right)<\log_{\sqrt{\frac{2x+3}{x-1}}}\left(\sqrt{7}+2\right).$ Answer: $(-4;-1{,}5].$

39). $\log_{\sqrt{\frac{x-6}{x-1}}}\sqrt{20}<\log_{\sqrt{\frac{x-6}{x-1}}}(2\pi-3).$ Answer: $(6;+\infty).$

40). $\log_{\sqrt{\frac{2x+1}{x-5}}}\left(\sqrt{7}+\sqrt{6}\right)<\log_{\sqrt{\frac{2x+1}{x-5}}}\left(\sqrt{8}+\sqrt{5}\right).$ Answer: $(-4;-0{,}5].$

41). $\log_{\sqrt{\frac{2x-1}{x-1}}}\sqrt{26}<\log_{\sqrt{\frac{2x-1}{x-1}}}(2\pi-2).$ Answer: $(0;0{,}5].$

42). $\log_{\sqrt{\frac{2x-7}{x-2}}}\left(\sqrt{5}+\sqrt{7}\right)<\log_{\sqrt{\frac{2x-7}{x-2}}}\left(\sqrt{3}+3\right).$ Answer: $(3{,}5;5).$

43). $\log_{\sqrt{\frac{2x-5}{x-1}}}\left(\sqrt{3}+2\right)<\log_{\sqrt{\frac{2x-5}{x-1}}}\left(\sqrt{2}+\sqrt{5}\right).$ Answer: $(2{,}5;4).$

44). $\sqrt{\dfrac{2x-1}{x+2}}-\sqrt{\dfrac{x+2}{2x-1}}\ge\dfrac{7}{12}.$ Answer: $(-\infty;-2)\cup[20{,}5;+\infty).$

45). $x^2+5x+4<5\sqrt{x^2+5x+28}.$ Answer: $(-9;4).$

46). $3^x\le 81.$ Answer: $(-\infty;4).$

47). $2^{9x-x^2}<1.$ Answer: $[-3;0]\cup[3;+\infty).$

48). $16^{\frac{2x+1}{3x-7}}-64^{\frac{1}{3}}\cdot(0{,}25)^{-2}>0.$ Answer: $\left(2\dfrac{1}{3};4{,}6\right).$

49). $\dfrac{4^x-2^{x+1}+8}{2^{1-x}}<8^x.$ Answer: $(1;+\infty).$

50). $\left(4x^2+2x+1\right)^{x^2-x}>1.$ Answer: $(-\infty;-0{,}5)\cup(1;+\infty).$

51). $\begin{cases}0{,}2^{\cos x}\le 1,\\ \dfrac{x-1}{2-x}+0{,}5>0.\end{cases}$ Answer: $\left(0;\dfrac{\pi}{2}\right].$

52). $(0{,}5)^{x^2-x}>3.$ Answer: $\varnothing.$

53). $(x-1)^{x^2-6x+8}>1.$ Answer: $(4;+\infty).$

54). $(0{,}5)^{\frac{x-4}{x}} \geq 8.$ Answer: $(-\infty;-4] \cup (0;1].$

55). $5^{2x+1} + 4 \cdot 5^{x} - 1 \geq 0.$ Answer: $(1;+\infty).$

56). $\log_{0,2}(x^2 - 2x - 3) \geq -1.$ Answer: $[-2;1) \cup (3;4].$

57). $\log_2(x^2 + x - 2) < \log_2(2x + 10).$ Answer: $(-3;-2) \cup (1;4).$

58). $\log_3 \frac{x+5}{x+1} < 0.$ Answer: $(-\infty;-5) \cup (-1;+\infty).$

Find the largest integer solution to an inequality:

$25^{x} - 2^{\log_4 6 - 1} < 10 \cdot 5^{x-1}.$ Answer: $\{0\}.$