Section 3

Exponentiation. Roots. Logarithms

Degree of number *a* with natural exponent  It is the product of *n* multipliers, each of which is equal to *a*.





The degree of an integral number *a* with an integral real exponent L=L0, L1 L2 L3 L4… is called the limit of the sequence of powers of the number *a* with rational exponents, which are approximate values of the number *L* up to 0,1; 0,01; 0,001… with a shortage. ****

By definition, we have: а =а; а   

For example: 51=5; 60=1;  

Properties of powers with an arbitrary real exponent:

1).  *multiplying degrees with identical bases;*

2).  *divisions of degrees with the same bases;*

3).  *exponentiation;*

4).  *raising the product to the degree;*

5).  *raising fractions.*

Find the value of an expression:

**3.1** а). 

b). 

c). 

d). 

e). 

f). 

g). 

**3.2** Simplify an expression and compute its value at *а*=1,8; *в*=0,5



Если а=1,8; в=0,5, то  Answer: 

**Self-study assignments:**

**3.3** Calculate:  Answer: 3.

Simplify expression: **3.4**  Answer: *х.*

**3.5**  Answer: 4.

**3.6**  Answer:  at 

**3.7**  Answer:  at 

 **3.8**  Answer: at *х*

**3.9**  Answer:  at 

**3.10**  Answer: –1 at 

**3.11** Given: 

To find  Answer: 

**3.12** Given: 

To find  Answer: 

**3.13** Given: 

To find  Answer: 

**3.14** 

Answer:  at 

**3.15**  Answer: 1 at 

**3.16**  Answer:  at 

**3.17**  Answer: 1 at 

Definition and properties of a root of a number

 The root of n-th degree  number *a* is called the number *b* , *п-о*й степень которого равна *а* . 

For example,   root exponent, 8 – root expression, 2 – root value.

The definition of a root implies the identity or 

If *n* - *is an even number*, there are two values of the square root of any positive number.

For example:  If *п –* *odd number*, then there is only one value of the root from any real number.

For example: 

Arithmetic root *n - degree*  with a positive number *a* is called an integral number *в*, *n - th* degree is equal to *а*.

For example: 

Finding the root of *n* - degree is called a root extraction.

**Properties arithmetic root:**

 the root of the product;

 root of the particle;

 root from root.



 If then 



  Here 2*т* – even number, *2т+1* – odd number.

We will show the application of the definition of a root of a number and its properties to solving exercises.

Find the value of an expression  Solution:

Decompose the number 75 by two factors so that at least one of them can be root 75=25 Number 48 decompose into two factors so that one of them is equal to 3: 48=3  We use the property of the root from the product: 

Answer:60. 

Solution: turn each of the "mixed" numbers into an improper fraction, and then simplify the radical expression:  Answer: 2.

 Solution: not one of 0,27, not one of (–100) the cube root is not extracted, and therefore we transform the radical expression in this way:



 Answer: –3.

 Solution: transform the radical expression using the formula for the difference of squares of two numbers:



 Answer: 2.

 Solution:

It is advisable to first get rid of irrationality in the denominator of each of the three fractions: 



 

. Answer: 33.

Noteworthy is this way of simplifying expressions like:

 Solution: denote 

Let us square both sides of this equality:

6+2

12-2

12-2;

12-2

А

А= Because the  then the difference 

Well then, А=2. Answer: 2.

Calculate the value of expression: **3.18** at *а =2,5.*

Solution:

Let's simplify this expression by reducing the radical expressions to a common denominator. Then we use the formula for the difference of squares and after cancellation we apply the concept of the modulus of the number.



If *а* = 2,5, then 

Answer: 2.

**3.19** Prove that 

Evidence:

 L.S. 

R.S. =  

Since the left and right sides are equal to the same numeric  then the equality is proved.

In the next two exercises, we will use the idea of raising to some power a selected part of an equality and simultaneously extracting a root from it of the same power..

**3.20** Prove equality 

Evidence:

Since the traditional getting rid of irrationality in the denominator of the fraction in this exercise does not lead to the goal, we perform the following transformations on the left side of the equality:

1. Erect it into a cube;
2. At the same time, we extract a cubic root from it.

 L.S. 

**3.21** Prove equality 

Evidence:

Since the left side of the expression is positive, to transform it we use the idea of solving the previous exercise:

L.S.

**Self-study assignments:**

Find the value of an expression:

**3.22**  Answer: 20.

**3.23 ** Answer: 30.

**3.24**  Answer: 2.

**3.25**  Answer: 5.

**3.26**  Answer: 0.

**3.27**  Answer: 8.

**3.28**  Answer: 2.

**3.29**  Answer: 0.

**3.30** Simplify expression:  Answer: 2.

**3.31** Find the value of an expression:  at *х*=26. Answer: 1.

**3.32** Simplify expression: 

Prove Equality:

**3.33** 

**3.34** 

**3.35** 

To find:

**3.36** *у* = if  Answer: *а+в.*

**3.37** *у=*  if , where   

Answer: 

**3.38** *у=* if  where 

Answer:  if  if 

**3.39**  if  where 

Answer: 

Определение и свойства логарифма числа

Logarithm of the number *N* with a base *а*  is the exponent to which you need to raise the number *a* to get the number *N.*

Depending on their base, logarithms can be classified into three large groups:



**Useful ratios:** *logaa=1;* *logа1=0.*

**Main logarithmic identity:** *alogaB =* *B.*

**Logarithms exist only for positive numbers and have the following properties:**

1). *loga*

2). 

3). 

4). 

5). 

6). 

Expressing the logarithm in terms of the logarithms of its components is called the logarithm. The inverse of the logarithm is called potentiation.

At first, it is advisable to solve exercises on the application of the basic logarithmic identity, which are good propaedeutics for solving logarithmic equations and inequalities.

For example: 







 Solution of more complex exercises of this type, for example, to calculate the value of the expression: 

 Solution:

We calculate the value of each of the three members of this expression separately:







Means, 25+5 – 6=24. Answer: 24.

**3.40** Calculate: 

 Solution:

We use the formula for the transition from one base of logarithms to another:

  In this way,

 Answer: 15.

**3.41** Calculate: 

Solution

Replace the amount of the logarithm logarithms product according to the properties of logarithms:

From the formula of the nth term of arithmetic progression 

find the number of its members: 

*an=a1+d(n-1); a1=1; a2=3; an=89.* 

*d=3-1=2.*

*89=1+2(n-1); 2(n-1)=89-1; 2(n-1)=88;*

*n-1= n=44+1=45.* Answer: 2025.

To calculate the sum of the first n terms of the arithmetic progression, we use the formula:



**3.42** Given:   To find 

Solution:

Let us transform the required logarithm: Answer: 

**3.43** Given: log To find 

Solution:

Expression  replace the logarithm with the base 6: 

Answer: 2.

**3.44** Given:  To find 

Solution:

 Let's move on to logarithms with base 12: 18=  

  

  

  

Answer: 

**3.45** Given:  To find 

Solution:

Let's move on to logarithms with base 20:



 

 

 

 

 Answer: 

**Self-study assignments:**

**3.46** It is known that *lg2* = 0,301. Calculate 

Answer: 0,301.

Calculate:

**3.47** 20*log* Answer:15.

**3.48** 18 Answer: 6

**3.49**  Answer: 11.

**3.50**  Answer: 19.

**3.51**  Answer: 2.

**3.52**  Answer: 9.

**3.53**  Answer: 6.

 **3.54**  Answer: 11.

**3.55**  Answer: 1,5.

**3.56**  Answer: 3,5.

**3.57** *в,* if *в=sіп*. Answer: –.

**3.58** 3 Answer:9.

**3.59**  если  Answer: 8.

**3.60**  Answer: 1,5

**3.61** 8+ Answer: 10.

**3.62**  если   Answer: 1,2.

**3.63**  если  Answer: 4.

**3.64**  Answer: 2.

To find *х,* if:

**3.65**  Answer: 

**3.66**  Answer: 

**3.67**  Answer: 11,25.

**3.68 G**iven:   To find  Answer: 

**3.69** Given:  To find  Answer: 

**3.70** Given:   To find  Answer: 

**3.71** Given:  To find  Answer: 

**3.72** Given:  To find  Answer: 

**3.73** To find  if  Answer: 

**3.74** To find  if  Answer: 

**3.75** To find  if  Answer: 

**3.76** To find  if  Answer: 

**3.77** To find  if  Answer: 